

Constraints on just enough inflation preceded by a thermal eraSuratna Das^{*}*Indian Institute of Technology, Kanpur 208016, India*Gaurav Goswami[†]*Institute of Engineering and Technology, Ahmedabad University, Navrangpura, Ahmedabad 380009, India*Jayanti Prasad[‡]*Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411007, India**Centre for Modeling and Simulation, Savitribai Phule Pune University, Pune 411007, India*Raghavan Rangarajan[§]*Theoretical Physics Division, Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India*

(Received 9 July 2015; published 22 January 2016)

If the inflationary era is preceded by a radiation-dominated era in which the inflaton too was in thermal equilibrium at some very early time, then the CMB data places an upper bound on the comoving temperature of the (decoupled) inflaton quanta. In addition, if one considers models of “just enough” inflation, where the number of e-foldings of inflation is just enough to solve the horizon and flatness problems, then we get a lower bound on the Hubble parameter during inflation, H_{inf} , which is in severe conflict with the upper bound from tensor perturbations. Alternatively, imposing the upper bound on H_{inf} implies that such scenarios are compatible with the data only if the number of relativistic degrees of freedom in the thermal bath in the preinflationary Universe is extremely large (greater than 10^9 or 10^{11}). We are not aware of scenarios in which this can be satisfied.

DOI: 10.1103/PhysRevD.93.023516

I. INTRODUCTION

The era in the early history of the Universe before cosmic inflation is completely unknown. It has been argued [1–3] that before the era of observable inflation, the Universe could have been sitting at a local minimum (metastable vacuum) in the effective potential of the underlying microscopic theory and then tunneled (via bubble nucleation) to a sufficiently flat direction causing slow-roll inflation. The standard thermal history of the Universe begins after the end of inflation, when the process usually referred to as reheating causes the Universe to get into a thermal state.

In this work, we consider a different picture of the early Universe. In our picture, the preinflationary early Universe is in a thermal radiation-dominated state, but as the Universe expands, the density of radiation decreases and eventually falls below the energy density corresponding to the rolling field, just like the energy density of the Universe gets dominated by a nonzero cosmological constant or quintessence field at late times. We ask what the present

cosmic microwave background (CMB) data can tell us about this possibility.

One crucial consequence of a preinflationary thermal history is that the quanta of the inflaton fluctuations $\delta\phi$ (the excitations about the classical inflaton background) could have been in thermal equilibrium very early in the preinflationary radiation era. If so, it would then subsequently decouple but would retain a thermal Bose-Einstein distribution even after decoupling. Hence, the field $\delta\phi$ during inflation would be in a thermal state [$\langle a_{\mathbf{k}}^\dagger a_{\mathbf{k}'} \rangle = [\exp(k/T) - 1]^{-1} \delta^3(\mathbf{k} - \mathbf{k}')$, where T is the inflaton comoving temperature], rather than in the Bunch-Davies vacuum state. This effect was studied in Ref. [4], where it was found that the primordial scalar power spectrum gets an additional factor of $\coth[k/(2T)]$ and the CMB temperature anisotropies get enhanced at large angular scales by an amount which depends upon the comoving temperature of the inflaton quanta during inflation.¹ From the measurements of temperature anisotropies of the CMB, one can put an upper limit on the comoving temperature of the inflaton. This then has

^{*} suratna@iitk.ac.in[†] gaurav.goswami@ahduni.edu.in[‡] jayanti@iucaa.ernet.in[§] raghavan@prl.res.in¹In Refs. [5–7], a thermal state of decoupled gravitons was considered which enhances the temperature anisotropy and the B-mode polarization angular power spectrum of the CMB at large angles.

implications for the duration of inflation [4]. A more detailed analysis, using COSMOMC, and including the modification of the inflaton mode functions due to a preinflationary radiation era, has been carried out by us in Ref. [8]. However, in Ref. [8] we treat both the comoving temperature T and the number of e-foldings of inflation more than that required to solve the horizon and flatness problems (δN) as independent parameters. We obtain a stronger constraint on the comoving temperature than in Ref. [4]. Furthermore, we find that the best-fit value of the number of e-foldings of inflation is very close to the minimum duration of inflation required to solve the horizon and flatness problems (though the improvement in likelihood is not very much over the standard power law spectrum).

Other previous studies of a scenario with a preinflationary radiation-dominated era have also shown that the CMB data favor scenarios with just enough inflation to solve the horizon and flatness problems (because of the low power on large scales in the TT angular power spectrum) [9–16] (see, however, Ref. [17]). We point out that there also exist good theoretical reasons for considering models of “just enough inflation” in which δN is small. By considering anthropic bounds on the value of the curvature and including certain statistical arguments on the parameters of inflation, it is argued in Ref. [1] that the number of e-foldings of inflation should not be much larger than the minimum required by observations. In Ref. [18] it is shown that by considering a certain natural canonical measure on the space of all classical universes, the probability for N e-foldings of inflation is suppressed by a factor of $\exp(-3N)$. Moreover, more inflation means that the inflaton potential is required to stay flat for a huge range of field values and can require super-Planckian excursions. Therefore, below we shall consider a scenario of just enough inflation.

None of the studies in Refs. [9–17] consider the possibility that the inflaton too could have been in thermal equilibrium during the preinflationary radiation era, as has been discussed above. In the scenario of Ref. [8] we presume that the inflaton was in thermal equilibrium at some early time in the preinflationary radiation era and that it subsequently went out of equilibrium and decoupled, and so include a (frozen) thermal Bose-Einstein distribution for the quanta of the inflaton field. We believe this is a natural extension of scenarios with a preinflationary radiation-dominated era. After all, the inflaton must have some couplings to other fields—some are needed for sufficient reheating to occur after inflation.

Our current analysis below shows that such a scenario with just enough inflation and a frozen thermal distribution for the inflaton quanta is compatible with the CMB data only if the effective number of relativistic degrees of freedom for the preinflationary radiation plasma is extremely large—in fact, larger than 10^9 or 10^{11} ,

depending on whether or not any entropy is transferred to the radiation after the inflaton decouples. This had not been recognized in Ref. [8].

II. THE MODEL

Previous studies of the preinflationary radiation era had considered either modifications of the mode functions of the inflationary scalar perturbations due to presence of the prior radiation era [9–17,19,20], or a thermal distribution of inflaton fluctuations associated with the inflaton being in thermal equilibrium at some very early phase in the radiation era [4]. While the first contribution lowers the quadrupole moment of the CMB TT anisotropy spectrum, the other contribution enhances the power at large angular scales. We, in Ref. [8], included both these effects as a consistent approach to determining the signatures of a preinflationary radiation era. We obtained the scalar power spectrum in Ref. [8] as

$$\mathcal{P}_{\mathcal{R}}(k) = A \left(\frac{k}{k_p} \right)^{n_s-1} \coth \left(\frac{k}{2T} \right) \mathcal{C}(k), \quad (1)$$

where $\mathcal{C}(k)$ reflects the modification of the inflaton mode functions, and hence the power spectrum, due to a preinflationary radiation era. The modified mode functions are obtained by setting Bunch-Davies initial conditions on the inflaton field in the preinflationary radiation-dominated era, and matching solutions for the field fluctuations and their time derivatives at the (instantaneous) transition from the radiation to the inflationary era (as opposed to simply setting the Bunch-Davies initial conditions in the inflationary era). Denoting the physical temperature of the inflaton by $\mathcal{T}(t)$, the comoving temperature $T = a(t)\mathcal{T}(t)$, with $a = 1$ today. The $\coth(\frac{k}{2T})$ factor appears due to the thermal distribution of the inflaton fluctuations: $1 + 2n_k = 1 + 2[\exp(k/T) - 1]^{-1} = \coth(\frac{k}{2T})$. If one considers only the modification of the inflationary mode function due to the presence of a preinflationary radiation era [10–17, 19,20], then the power spectrum would not contain the $\coth[k/(2T)]$ factor; and similarly, considering only the thermal distribution of the inflaton quanta, the power spectrum would not contain the $\mathcal{C}(k)$ factor [4].

We carried out a detailed COSMOMC analysis in Ref. [8], using Planck 2013 and WMAP nine-year data and the power spectrum in Eq. (1). In addition to the six standard parameters for the Monte Carlo simulation we included two extra parameters— δN , the number of e-foldings of inflation in excess of the minimum required to solve the horizon and flatness problems, and T , the comoving temperature. [The factor $\mathcal{C}(k)$ in the power spectrum depends on δN through the time when the change from the radiation era to the inflationary era takes place.] For an inflationary scale of 10^{15} GeV we found that the best-fit value of δN is 0.08 (the marginalized value is consistent with zero with

68% confidence). This indicates that the data favor a “just enough” inflationary scenario. As discussed earlier, there are also theoretical motivations for considering such a scenario. We also obtained an upper limit on the comoving temperature as $T < 1.3 \times 10^{-4} \text{ Mpc}^{-1}$ at a 68% C.L. (for the GUT and the electroweak scale).

We will now show that this scenario with a frozen thermal distribution of inflaton fluctuations and “just enough” inflation preceded by a radiation-dominated era faces a severe conflict, as it either implies a very large Hubble parameter during inflation that conflicts with the known upper bound, or requires an extremely large number of relativistic degrees of freedom (greater than 10^9 or 10^{11}) in the early Universe.

III. ANALYSIS

In the following analysis, we use the subscript 0 to refer to the present epoch. Let t_* be the epoch when the mode k_0 corresponding to our current Hubble radius was crossing the Hubble radius during inflation. Then

$$k_0 = a_* H_* = a_0 H_0, \quad (2)$$

where $a_* = a(t_*)$, $H_* = H(t_*)$, $H(t_0) = H_0$, and $a_0 = 1$. Presuming that the Hubble parameter is approximately constant during inflation, we set $H_* = H_{\text{inf}} = M_{\text{inf}}^2 / (\sqrt{3} M_{\text{P}})$, where M_{inf} and $M_{\text{P}} = 2.4 \times 10^{18} \text{ GeV}$ are the energy scale of inflation and the reduced Planck mass, respectively. Then

$$\begin{aligned} \mathcal{T}_* &= T/a_* \\ &= \frac{TH_{\text{inf}}}{H_0}. \end{aligned} \quad (3)$$

If there were δN e-foldings of inflation before t_* , then

$$\mathcal{T}_i = \mathcal{T}_* \exp \delta N, \quad (4)$$

where \mathcal{T}_i is the physical inflaton temperature at the beginning of inflation.

We assume that the radiation energy density at the onset of inflation at t_i is equal to that of the inflaton, i.e. $\rho_r = \rho_\phi$. Then the radiation temperature at t_i is

$$\mathcal{T}_\gamma = \left(\frac{30}{\pi^2 g_{\text{rel}}} \right)^{1/4} M_{\text{inf}} \quad (5)$$

$$= \left(\frac{90}{\pi^2 g_{\text{rel}}} \right)^{1/4} \sqrt{H_{\text{inf}} M_{\text{P}}}. \quad (6)$$

Here g_{rel} is the effective number of relativistic degrees of freedom for the plasma filling the Universe at the onset of inflation. If the underlying microscopic theory is the Standard Model of elementary particle physics, $g_{\text{rel}} = 106.75$, while if it is MSSM, then $g_{\text{rel}} = 228.75$

[21]. If the number of relativistic degrees of freedom stays the same between the epoch of decoupling of the inflaton and the epoch of onset of inflation (i.e. no species annihilates or becomes nonrelativistic in between), we can assume that the inflaton temperature equals the radiation temperature at the onset of inflation, i.e.

$$\mathcal{T}_i = \mathcal{T}_\gamma. \quad (7)$$

Then, combining Eqs. (3), (4), (6) and (7), we get

$$\frac{H_{\text{inf}}}{M_{\text{P}}} = \left(\frac{90}{\pi^2 g_{\text{rel}}} \right)^{1/2} \left(\frac{H_0}{T} \right)^2 e^{-2\delta N}. \quad (8)$$

Below, we shall take

$$T < 1 \times 10^{-4} \text{ Mpc}^{-1}. \quad (9)$$

As mentioned above, for GUT scale inflation the best-fit value of δN is 0.08. Assuming the energy scale to be much smaller (e.g. the electroweak scale) implies that the best fit δN is 0.02. If we assume that $\delta N \approx 0.05$ and $T < 10^{-4} \text{ Mpc}^{-1}$, and take $H_0 \approx (4400 \text{ Mpc})^{-1}$ [22] and $g_{\text{rel}} \sim 100$, then Eq. (8) implies that

$$\frac{H_{\text{inf}}}{M_{\text{P}}} > 1, \quad (10)$$

or

$$M_{\text{inf}} > M_{\text{P}}. \quad (11)$$

But from the upper bound on r at $k = 0.002 \text{ Mpc}^{-1}$ (viz. $r < 0.1$), we have [23]

$$\frac{H_{\text{inf}}}{M_{\text{P}}} < 3.7 \times 10^{-5}, \quad (12)$$

or $M_{\text{inf}} < 1.9 \times 10^{16} \text{ GeV}$. Clearly the lower bound on H_{inf} in Eq. (10) is in severe conflict with the CMB constraint. [Note that for $k = 0.002 \text{ Mpc}^{-1}$ the coth factor and \mathcal{C} in Eq. (1) reduce to ~ 1 , and hence we can use Eq. (12).]

As is clear from Eq. (8), this result is a consequence of picking a small value of δN [larger values of $\delta N \gtrsim 6$ will decrease the lower bound on H_{inf} in Eq. (10) to acceptable levels]. Our result is thus a serious constraint on inflationary models with just enough inflation which are preceded with a preinflationary radiation era where one assumes that the inflaton too could have been in thermal equilibrium at some early time prior to inflation.

We now study possible solutions to this conundrum. Equation (8) implies that Eq. (10) can be recast as

$$\frac{H_{\text{inf}}}{M_{\text{P}}} > \frac{10}{\sqrt{g_{\text{rel}}}}. \quad (13)$$

However, Eq. (12) then implies that

$$g_{\text{rel}} > 10^{11}. \quad (14)$$

This is an extremely large number of relativistic degrees of freedom in the early Universe.

At this point it is worthwhile recalling another assumption we have made in arriving at our result. As Eq. (7) indicates, we have assumed that the temperature of the decoupled inflaton quanta and that of the preinflationary radiation plasma is the same. This assumption will break down if between the epoch of decoupling of the inflaton (at time t_{dec}) and the onset of inflation (at time t_i) some species annihilates and hence causes the temperature of the radiation to be more than that of the inflaton at t_i . Let us see what are the consequences of relaxing the assumption of Eq. (7).

We now suppose that at the Planck time, the inflaton, BGUT particles, and GUT particles were in thermal equilibrium (BGUT refers to Beyond GUT), possibly via gravitational interactions. After the Planck scale, at time t_{dec} , the inflaton decouples. A bit later, at a time t_1 , the BGUT particles become nonrelativistic and annihilate in equilibrium and transfer their entropy to the GUT particles but not to the inflaton (since it is already decoupled). Using b and a for before and after (instantaneous) annihilation, conservation of entropy implies that

$$\frac{\mathcal{T}_b}{\mathcal{T}_a} = \left(\frac{g_{*a}}{g_{*b}}\right)^{\frac{1}{3}}, \quad (15)$$

where g_{*b} is the number of relativistic degrees of freedom before the annihilation and g_{*a} is the number of relativistic degrees after the annihilation.

Assuming no further significant entropy production from t_1 till t_i , the temperature of the inflaton quanta at the onset of inflation is

$$\mathcal{T}_i = \mathcal{T}_b(a_1/a_i), \quad (16)$$

while the temperature of the radiation evolves to

$$\mathcal{T}_\gamma = \mathcal{T}_a(a_1/a_i) \quad (17)$$

$$= \mathcal{T}_b \left(\frac{g_{*b}}{g_{*a}}\right)^{\frac{1}{3}} (a_1/a_i) \quad (18)$$

and $g_{\text{rel}} = g_{*a}$. Then

$$\mathcal{T}_i = (\mathcal{T}_b/\mathcal{T}_a)\mathcal{T}_\gamma, \quad (19)$$

and using Eq. (15) and the fact that $\rho_\phi = \rho_\gamma$ at the beginning of inflation, we get

$$\mathcal{T}_i = \left(\frac{g_{*a}}{g_{*b}}\right)^{\frac{1}{3}} \left(\frac{90}{\pi^2 g_{*a}}\right)^{\frac{1}{4}} \sqrt{H_{\text{inf}} M_{\text{P}}}. \quad (20)$$

Combining Eq. (20) with Eqs. (3) and (4), and using the bounds in Eqs. (9) and (12), and setting $\delta N = 0.05$ and $H_0 \approx (4400 \text{ Mpc})^{-1}$ gives

$$g_{*b} > 2 \times 10^8 g_{*a}^{1/4} \approx 10^9. \quad (21)$$

Such a large change in g_* cannot be obtained from the annihilation of a single species. However, if multiple species annihilate in equilibrium at t_1, t_2, \dots, t_n and transfer their entropy to radiation, then

$$\mathcal{T}_i = \mathcal{T}_{b1}(a_1/a_i), \quad (22)$$

$$\begin{aligned} \mathcal{T}_\gamma &= \mathcal{T}_{a1} \left(\frac{\tau_{a2}}{\tau_{b2}}\right) \dots \left(\frac{\tau_{an}}{\tau_{bn}}\right) \left(\frac{a_1}{a_2}\right) \left(\frac{a_2}{a_3}\right) \dots \left(\frac{a_n}{a_i}\right) \\ &= \mathcal{T}_{b1} \left(\frac{g_{*b1}}{g_{*a1}}\right)^{\frac{1}{3}} \left(\frac{g_{*b2}}{g_{*a2}}\right)^{\frac{1}{3}} \dots \left(\frac{g_{*bn}}{g_{*an}}\right)^{\frac{1}{3}} \left(\frac{a_1}{a_i}\right) \\ &= \mathcal{T}_{b1} \left(\frac{g_{*b1}}{g_{*an}}\right)^{\frac{1}{3}} (a_1/a_i), \end{aligned} \quad (23)$$

as in Eq. (17) with $g_{*an} = g_{\text{rel}}$. ($g_{*bm+1} = g_{*am}$.) Then we obtain the same result as in Eq. (21) for g_{*b1} , i.e. the number of relativistic degrees of freedom before the annihilations must be greater than 10^9 .

We thus conclude that any scenario of just enough inflation, in which the preinflationary universe was radiation dominated and in which the inflaton quanta too were in thermal equilibrium at some early epoch, requires an extremely large number of relativistic species to be present in the preinflationary radiation plasma to be consistent with the CMB data.

IV. DISCUSSION AND CONCLUSION

We now present a physical interpretation of our results. The $\coth[k/(2T)]$ factor in our power spectrum leads to an enhancement of power on large scales. To suppress this requires putting an upper bound on the comoving temperature T . If we express the \coth factor for the current horizon scale as $\coth[k_0/(2a_* T_*)] = \coth[H_{\text{inf}}/(2T_*)]$, then the upper bound on T translates into an upper bound on T_* . For a small value of δN , i.e., if the duration of inflation is just enough to solve the horizon and flatness problems, $\mathcal{T}_i \approx T_*$. Now if the (frozen) inflaton temperature at the beginning of inflation is the same as that of the thermal radiation in the preinflationary radiation era, then one gets an upper bound on the radiation temperature

$$\mathcal{T}_\gamma < 4 \times 10^{13} \text{ GeV} \frac{H_{\text{inf}}}{9 \times 10^{13} \text{ GeV}}. \quad (24)$$

To satisfy this low bound on the preinflationary radiation temperature and the condition $\rho_r = \rho_\phi = 3H_{\text{inf}}^2 M_{\text{P}}^2$ at the beginning of inflation then requires g_{rel} to be larger

than 10^{11} . Alternatively, the radiation temperature at the beginning of inflation can be much larger than the inflaton temperature if entropy was released in the Universe after the inflaton decoupled. Then, while \mathcal{T}_i will be less than 4×10^{13} GeV, T_γ can be as high as 1×10^{16} GeV with $g_{\text{rel}} \sim 100$. But then g_* before entropy release is $(\mathcal{T}_\gamma/\mathcal{T}_i)^3 g_{\text{rel}} \approx 10^9$.

In models of thermal inflation invoked to solve the moduli problem [24–26], primordial inflation is followed by a later phase of “thermal” inflation which is driven by the potential energy of a “flaton” field trapped at the origin at the minimum of its thermal effective potential [24,25,27]. Density perturbations generated during the short later phase of thermal inflation influence the spectrum only on very small scales, while the larger-scale perturbations which influence the large-scale structure and cosmic microwave background are largely generated during the earlier primordial inflation [28]. Since the flaton has thermal interactions one would expect that the flaton quanta would have a thermal Bose-Einstein distribution, which has so far not been considered. In models of warm inflation a thermal bath is generated by dissipation of the inflaton field and is maintained in the Universe during inflation. If the inflaton fluctuations are also in thermal equilibrium with the thermal bath there will again be a coth term in the density perturbation spectrum, as seen, for example, in Refs. [29–31]. In both thermal and warm inflation, the presence of the coth term would affect density perturbations with large power on the relevant large scales, as in Ref. [4], and can also lead to further constraints as discussed above if one also imposes a finite duration of inflation. We plan to study these scenarios in greater detail in the future.

In conclusion, we have explored a scenario of inflation preceded by a radiation-dominated era. We presume that the inflaton too could have been in thermal equilibrium

with the radiation at some very early era, and so we consider a frozen thermal state for the inflaton field with a Bose-Einstein distribution for the (decoupled) inflaton quanta. There is then an upper bound on the comoving relic temperature for the inflaton [4,8]. Then, in these inflation scenarios and with just enough e-foldings to solve the horizon and flatness problem, which is motivated by constraints on large inflaton field excursions and is also indicated by the CMB data, we find a lower bound on the Hubble parameter during inflation which is in severe conflict with the upper bound on the Hubble parameter from CMB observations. To resolve this conflict one has to allow for an extremely large number of relativistic degrees of freedom in the thermal bath in the very early Universe, greater than 10^9 or 10^{11} depending on whether or not there has been entropy transfer to radiation after the inflaton decoupled. We are not aware of any scenario that may give such large values of g_* in the early Universe.

ACKNOWLEDGMENTS

R. R. would like to thank S. Minwalla and S. Trivedi for helpful discussions. We would like to thank the first referee for constructive comments about our work that helped to improve the presentation of our results. We would also like to thank the second referee for the suggestion to apply our analysis to models of thermal and warm inflation. Numerical work for the present study was done at the IUCAA HPC facility. Work of S. D. is supported by the Department of Science and Technology, Government of India under Grant Agreement No. IFA13-PH-77 (INSPIRE Faculty Award). J. P. would like to thank the Science and Engineering Research Board (SERB) of the Government of India for financial support via a Start-Up Research Grant (Young Scientists) No. SR/FTP/PS-102/2012.

-
- [1] B. Freivogel, M. Kleban, M. Rodriguez Martinez, and L. Susskind, *J. High Energy Phys.* **03** (2006) 039.
 - [2] R. Bousso, D. Harlow, and L. Senatore, *Phys. Rev. D* **91**, 083527 (2015).
 - [3] R. Bousso, D. Harlow, and L. Senatore, *J. Cosmol. Astropart. Phys.* **12** (2014) 019.
 - [4] K. Bhattacharya, S. Mohanty, and R. Rangarajan, *Phys. Rev. Lett.* **96**, 121302 (2006).
 - [5] M. Gasperini, M. Giovannini, and G. Veneziano, *Phys. Rev. D* **48**, R439 (1993).
 - [6] K. Bhattacharya, S. Mohanty, and A. Nautiyal, *Phys. Rev. Lett.* **97**, 251301 (2006).
 - [7] W. Zhao, D. Baskaran, and P. Coles, *Phys. Lett. B* **680**, 411 (2009).
 - [8] S. Das, G. Goswami, J. Prasad, and R. Rangarajan, *J. Cosmol. Astropart. Phys.* **06** (2015) 001.
 - [9] J. M. Cline, P. Crotty, and J. Lesgourgues, *J. Cosmol. Astropart. Phys.* **09** (2003) 010.
 - [10] B. A. Powell and W. H. Kinney, *Phys. Rev. D* **76**, 063512 (2007).
 - [11] G. Nicholson and C. R. Contaldi, *J. Cosmol. Astropart. Phys.* **01** (2008) 002.
 - [12] G. Marozzi, M. Rinaldi, and R. Durrer, *Phys. Rev. D* **83**, 105017 (2011).
 - [13] M. Cicoli, S. Downes, B. Dutta, F. G. Pedro, and A. Westphal, *J. Cosmol. Astropart. Phys.* **12** (2014) 030.
 - [14] S. Hirai, *Classical Quantum Gravity* **22**, 1239 (2005).

- [15] S. Hirai and T. Takami, *Classical Quantum Gravity* **23**, 2541 (2006).
- [16] S. Hirai and T. Takami, [arXiv:0710.2385](https://arxiv.org/abs/0710.2385).
- [17] I.-C. Wang and K.-W. Ng, *Phys. Rev. D* **77**, 083501 (2008).
- [18] G. Gibbons and N. Turok, *Phys. Rev. D* **77**, 063516 (2008).
- [19] S. Hirai, *Classical Quantum Gravity* **20**, 1673 (2003).
- [20] S. Hirai, [arXiv:hep-th/0307237](https://arxiv.org/abs/hep-th/0307237).
- [21] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990).
- [22] P. Ade *et al.* (Planck Collaboration), *Astron. Astrophys.* **571**, A16 (2014).
- [23] P. Ade *et al.* (Planck Collaboration), *Astron. Astrophys.* **571**, A22 (2014).
- [24] D. H. Lyth and E. D. Stewart, *Phys. Rev. Lett.* **75**, 201 (1995).
- [25] D. H. Lyth and E. D. Stewart, *Phys. Rev. D* **53**, 1784 (1996).
- [26] L. Randall and S. D. Thomas, *Nucl. Phys.* **B449**, 229 (1995).
- [27] T. Hiramatsu, Y. Miyamoto, and J. Yokoyama, *J. Cosmol. Astropart. Phys.* **03** (2015) 024.
- [28] S. E. Hong, H.-J. Lee, Y. J. Lee, E. D. Stewart, and H. Zoe, *J. Cosmol. Astropart. Phys.* **06** (2015) 002.
- [29] R. O. Ramos and L. A. da Silva, *J. Cosmol. Astropart. Phys.* **03** (2013) 032.
- [30] S. Bartrum, M. Bastero-Gil, A. Berera, R. Cerezo, R. O. Ramos, and J. G. Rosa, *Phys. Lett. B* **732**, 116 (2014).
- [31] M. Bastero-Gil, A. Berera, I. G. Moss, and R. O. Ramos, *J. Cosmol. Astropart. Phys.* **05** (2014) 004.