Joint Planck and WMAP assessment of low CMB multipoles

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Abstract. The remarkable progress in cosmic microwave background (CMB) studies over past decade has led to the era of precision cosmology in striking agreement with the ΛCDM model. However, the lack of power in the CMB temperature anisotropies at large angular scales (low-ℓ), as has been confirmed by the recent Planck data also (up to $\ell = 40$), although statistically not very strong (less than 3σ), is still an open problem. One can avoid to seek an explanation for this problem by attributing the lack of power to cosmic variance or can look for explanations i.e., different inflationary potentials or initial conditions for inflation to begin with, non-trivial topology, ISW effect etc. Features in the primordial power spectrum (PPS) motivated by the early universe physics has been the most common solution to address this problem. In the present work we also follow this approach and consider a set of PPS which have features and constrain the parameters of those using WMAP 9 year and Planck data employing Markov-Chain Monte Carlo (MCMC) analysis. The prominent feature of all the models of PPS that we consider is an infra-red cut off which leads to suppression of power at large angular scales. We consider models of PPS with maximum three extra parameters and use Akaike information criterion (AIC) and Bayesian information criterion (BIC) of model selection to compare the models. For most models, we find good constraints for the cut off scale k_c, however, for other parameters our constraints are not that good. We find that sharp cut off model gives best likelihood value for the WMAP 9 year data, but is as good as power law model according to AIC. For the joint WMAP 9 + Planck data set, Starobinsky model is slightly preferred by AIC which is also able to produce CMB power suppression up to $\ell \leq 30$ to some extent. However, using BIC criteria, one finds model(s) with least number of parameters (power law model) are always preferred.

Keywords: cosmological parameters from CMBR, inflation, CMBR theory

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Contents

1 Introduction

Anisotropies in the temperature and polarization of cosmic microwave background (CMB) have information about the early universe and can be used to study the primordial fluctuations generated during inflation which lead to structure formation in the universe [1–6]. CMB anisotropies are believed to be sourced by quantum fluctuations generated during inflation [2, 3, 7, 8]. Thus by investigation of the CMB angular power spectrum, which completely characterizes CMB anisotropies, one is able to probe the physics of the very early universe.

So far most CMB experiments indicate that CMB anisotropies are statistically isotropic and Gaussian and so can be completely characterized by their two-point correlations or power spectrum [9–11]. Although, almost all the CMB observations confirm that the six parameter ΛCDM cosmological model best fits the observed data, still there are some anomalies which have always been present from COBE to Planck. One of such anomalies has been the lack of power in the CMB-TT power spectrum (C_l^{TT}) at large angular scales or low-ℓ [12–14]. Recently, [15] studied the consistency of the standard ΛCDM model with the Planck data using the Crossing statistic [16–18]. Their results indicate that the Planck data is consistent to the concordance ΛCDM only at 2-3σ confidence level and lack of power at both high and low ℓ's with respect to concordance model. The low power at large angular scales can be attributed to cosmic variance [19], still there have been efforts to explain this anomaly by changing the potential of inflation field (for a comprehensive review check [20]), considering different initial conditions at the beginning of inflation [21–28], ISW effect [29], spatial curvature [30], non-trivial topology [31], geometry [32, 33], violation of statistical anisotropies [34], cosmological-constant type dark energy during the inflation [35], bounce from contracting phase to inflation [36, 37], production of primordial micro black holes (MBH) remnants in the very early universe [38], hemispherical anisotropy and non-gaussianity [39, 40], string theory [41], loop quantum cosmology [42] etc. Although, inflationary ΛCDM model with

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almost scale-invariant power spectrum has emerged most successful model from the recent observations, it is important to note that it does not uniquely confirm the generic picture of the universe and the generalization of primordial power spectrum having additional features like cut off, oscillations would be crucial in identifying specific inflationary models.

There are two main approaches which have been followed to probe the primordial power spectrum of curvature fluctuations generated during inflation from the CMB anisotropies. In the first approach no specific model of PPS is considered and the shape of PPS is directly reconstructed from the data by deconvolution using different techniques [43–50]. The main disadvantage of this approach is that the angular power spectrum does not have all the information about PPS due to nature of transformation kernel and some form of regularization may be needed which penalizes models which have features not desired. Maximum Entropy method [51] has also been applied for this purpose. Planck team has used a form of regularization which penalizes any model of PPS which deviates from a straight line and has non-zero power at those scales at which there are no constrains [52]. The second approach which has been used to probe the primordial power spectrum from the CMB data has been to consider physically motivated models of the early universe, represented by a PPS with features, and estimate the parameters of those from CMB data [21, 23, 36, 53–55]. Some of the models of PPS are inspired from the change in physical conditions during inflation represented by change in the potential of inflation during slow roll or initial conditions at the beginning of inflation. In the present work, we consider a set of models of PPS which were studied in [55] and try to constrain the parameters of those from the WMAP 9 year and Planck data. We add a couple of other models also to our analysis. All the models we consider have a common feature that they all have an infrared cut off in power at large scales and match perfectly with the standard power law model at small scales.

The plan of this paper is as follows: in section 2, we give a brief outline of inflationary framework to introduce primordial power spectrum and its parameters and also give a short introduction of the models (PPS) we consider for our analysis. In section 3, we present the results of our analysis in the form of the best fit model parameters of PPS. We also give a comparison of our models using Akaike information criterion (AIC) and Bayesian information criterion (BIC) in this section. Discussion and conclusions of our work are given in the last section.

2 Inflationary fluctuations

Inflation is characterized by a phase in the early universe when the energy density of the universe is dominated by a scalar field ϕ that can be characterized by a perfect fluid with the diagonal components of the energy-momentum tensor given by the energy density ρ_ϕ and pressure density p_ϕ respectively [7, 8, 20].

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{2.1}$$

and

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{2.2}$$

where $V(\phi)$ is the potential energy of the scalar field.

The dynamics of the scalar field that leads to inflation is governed by the following equation (in FRW case):

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{2.3}$$

and

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{2.4}$$

Slow-roll inflation is characterized by two parameters ϵ and η which are defined as:

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \tag{2.5}$$

and

$$\eta = \epsilon + \delta = M_{\rm Pl}^2\left(\frac{V''(\phi)}{V(\phi)}\right), \tag{2.6}$$

where

$$\delta = -\frac{\ddot{\phi}}{H\dot{\phi}}. \tag{2.7}$$

For inflation to happen we need $\epsilon << 1$ and $|\eta| << 1$.

During inflation fluctuation $\delta\phi$ in the scalar field ϕ lead to fluctuation $\mathcal{R}$ in the spatial curvature which are characterized by their two point correlation function under common assumptions (homogeneity & isotropy):

$$\langle \mathcal{R}^*(\mathbf{k})\mathcal{R}(\mathbf{k}')\rangle \;=\; \delta^3(\mathbf{k}-\mathbf{k}')\Delta^2_{\mathcal{R}}(k)\,, \qquad \mathcal{P}_o(k) \equiv \frac{k^3}{2\pi^2}\,\Delta^2_{\mathcal{R}}(k)\,, \tag{2.8}$$

where the angular brackets denote an ensemble average, δ is the Dirac delta function and $\mathcal{P}_o(k)$ is called primordial scalar power spectrum. In the standard ΛCDM cosmology the shape of the primordial power spectrum in its simplest form can be expressed in power-law parameterization. This model is referred to as Power Law model and can be obtained at leading order slow-roll approximation of the single-inflation field [56]:

$$\mathcal{P}_o(k) = A_S\left(\frac{k}{k_0}\right)^{n_s-1}, \tag{2.9}$$

where n_s is called spectral index (tilt parameter) and is expected to be close to 1, A_s is spectral amplitude, k_0 is the scalar pivot which is set equal to $0.05\,\mathrm{Mpc}^{-1}$ throughout this work. The scalar primordial power spectrum parameters can be calculated in terms of slow roll parameters (ϵ, η) as

$$A_s \simeq \frac{V}{24\pi^2\epsilon M_P^4}, \quad n_s \;\simeq\; 1 + 2\eta - 6\epsilon. \tag{2.10}$$

In addition, to the scalar primordial power spectrum, inflation also predicts a tensor spectrum $\mathcal{P}_t(k)$ due to gravity-wave (tensor) perturbations which is usually written in the form

$$\ln\mathcal{P}_t(k) \;=\; \ln A_t \;+ n_t\ln\left(\frac{k}{k_0}\right), \tag{2.11}$$

where A_t and n_t are the tensor amplitude and spectral index respectively. In terms of slow roll parameter these can be written as

$$A_t \simeq \frac{3V}{2\pi^2 M_P^2}, \quad r \equiv \frac{\mathcal{P}_t}{\mathcal{P}_o} \simeq 16\epsilon, \quad n_t \simeq -\frac{r}{8}. \tag{2.12}$$

The major contribution to $P_t(k)$ comes form the B-mode polarization and in light of recent questions regarding the claims of the BICEP2 results [57–59], we will not use B-mode BICEP2 polarization data in our analysis and therefore assume $r = 0$ (or $P_t(k) = 0$) (i.e consider scalar perturbations only).

The inflation theory predict a temperature fluctuations to be statistically isotropic with very nearly Gaussian of zero mean, consistent with current observations. It is customary to represent theoretical and experiment temperature power spectrum in terms of spherical harmonics as [60]

$$\frac{\Delta T(\hat{n})}{T} = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{m=\ell} a_{\ell m} \, Y_{\ell m}(\hat{n}), \tag{2.13}$$

where $\hat{n} \equiv (\theta, \phi)$ is a unit direction vector on the sky, $a_{\ell m}$ are complex quantities and $Y_{\ell m}(\hat{n})$ are normalized spherical harmonics. Assuming CMB fluctuations to be Gaussian distributed, then each a_{lm} is independent with exception equal to zero and Gaussian distributed:

$$< a^*_{lm} a_{l'm'} >= \delta_{ll'} \, \delta_{mm'} \, C_l, \tag{2.14}$$

where C_l is called the angular power spectrum. In practice, CMB angular power spectrum C_ℓ is computed using the two-point angular correlation function

$$C(\hat{n}_1 \cdot \hat{n}_2) = \left\langle \frac{\Delta T}{T}(\hat{n}_1) \frac{\Delta T}{T}(\hat{n}_2) \right\rangle = \sum_{\ell=2}^{\infty} \frac{2\ell+1}{4\pi} \, C_\ell \, P_\ell \, (\hat{n}_1 \cdot \hat{n}_2) \, , \tag{2.15}$$

where P_ℓ is the Legendre polynomials. The measured angular power spectrum C_l is a robust cosmological probe in constraining cosmological models, the position and amplitude of the peaks being very sensitive to important cosmological parameters. Since Thomson scattering of an anisotropic radiation field also generates linear polarization [60], there are also angular power spectrum due to the polarization. The polarization anisotropies have a different dependence on cosmological parameters than that for temperature power spectrum and can provide a way to break degeneracies in various cosmological parameters. Moreover, since polarization data is free from ISW effect one can easily separate out the contribution of low CMB power due to ISW and infrared cut off. The polarization can be divided into parts that come from curl (B-mode) and divergence (E-mode) yielding four independent angular power spectra as C_ℓ^{TT},C_ℓ^{EE}, C_ℓ^{TE}, C_ℓ^{BB}. The initial power spectrum $\mathcal{P}(k)$ is related to the angular power spectrum C_ℓ through

$$C_\ell^{XX'} \propto \int \mathrm{dln}k \, \mathcal{P}(k) \, T_\ell^X(k) \, T_\ell^{X'}(k), \tag{2.16}$$

where $T_\ell^X(k)$ is the transfer function with X representing the CMB temperature or polarization.

One of the noteworthy outcomes from recent cosmological results, especially from WMAP and Planck, is the possibility of obtaining structural form of the primordial power spectrum [61–64], which in turn has potential to differentiate strongly between various inflationary models dominating early universe physics. The most commonly used primordial spectrum is almost scale-invariant power law during inflation which went on to produce the observed structure in the CMB. However, different inflationary models readily accommodate different primordial spectra with radical departures from this simple picture, especially at low k. Moreover, recent results of WMAP and Planck have confirmed the general picture of the primordial power spectrum having a suppression at low k which could not be explained by scale-invariant power law model. Deconvolution of CMB data strongly favors a cut off around horizon scale $0.00001\,\mathrm{Mpc}^{-1} < k_c < 0.0009\,\mathrm{Mpc}^{-1}$ followed by a bump in a primordial power spectrum [62, 63]. Motivated by the fact that primordial power spectra with a cut off should give better likelihood than scale free power law model, we will next point out various primordial power spectra which have cut off at low k arising due to the physics in the initial phase of inflationary models.

In the present work we consider models of primordial power spectrum (PPS) which suppress power at large scales (small-k) and agree at small scales with the standard power law model since our aim here is to explain the deficiency of power at large angular scales in CMB-TT power spectrum. By considering models with a large number of fitting parameters it becomes easier to fit the data, however, any method of model comparison must penalize models with a large number of fitting parameters. In the present work, we consider models with at the most three extra parameters of PPS (apart from two usual parameters A_s and n_s). For model selection we will use AIC and BIC in the next section.

2.1 Model 1: power law (PL)

We consider the standard power law power spectrum "$\mathcal{P}_o(k)$" characterized by two parameters, spectral index (n_s) and amplitude A_s at some pivot scale k_0. Since this model is a part of the standard six parameters cosmological model therefore we compare the improvement in the likelihood as compared to this model. $\mathcal{P}_0(k)$ is give by eq. (2.9) and can be rewritten as:

$$\ln \mathcal{P}(k) = \ln A_s + (n_s - 1) \ln \left(\frac{k}{k_0}\right). \tag{2.17}$$

All the models we consider in the present work can be written as modulation over the power law model:

$$\mathcal{P}(k) = \mathcal{P}_0(k) \times \mathcal{F}(k, \mathbf{\Theta}), \tag{2.18}$$

where $\mathcal{F}(k, \mathbf{\Theta})$ is the "modulation" part and $\mathbf{\Theta}$ is a vector which characterizes the extra parameters.

2.2 Model 2: running spectral index (RN)

Scale dependent spectral index n_s, as characterized by an extra parameter α_s called "running index", has been a part of the extension of the standard six parameter cosmological model and is well motivated in the inflationary framework [56, 65–67]. In the slow-roll approximation, α_s, is second order term and is of the order of 10^{-3} and therefore was assumed to be zero for the power law model discussed in previous section. α_s can be calculated in terms of slow roll parameters as

$$\alpha_s \simeq -2\xi + 16\epsilon\eta - 24\epsilon^2, \tag{2.19}$$

where the slow roll parameter ξ is related to the third derivative of the inflationary potential $V(\phi)$ in the following way

$$\xi = M_{\rm Pl}^4 \frac{V' V'''}{V^2}. \tag{2.20}$$

Although, larger value of α_s could produce suppression of CMB power, but sizable value of α_s will amount to violation of slow roll approximation. However, there are certain models [65–69] where α_s can be large, while still respecting the slow-roll approximation. Therefore, in our analysis we also consider a model with non-zero α_s.

Current CMB observations slightly favor a non-zero running model of PPS over the standard power law PPS and in the present work we try to constrain the parameter α_s with the WMAP 9 year and Planck data. We use the standard parameterization for the running which is given by the following equation.

$$\ln \mathcal{P}(k) \; = \; \ln A_s + (n_s - 1) \ln \left(\frac{k}{k_0} \right) + \frac{\alpha_s}{2} \left[\ln \left(\frac{k}{k_0} \right) \right]^2 . \tag{2.21}$$

2.3 Model 3: sharp cut off (SC)

This model assumes that there is sharp cut off in the primordial power spectrum at large scale:

$$P(k) = \begin{cases} A_s \left(\frac{k}{k_c} \right)^{n_s - 1}, & \text{for } \; k > k_c \\ 0, & \text{otherwise.} \end{cases}$$

This model was considered in [48, 70] and constraints were found for the cut off scale. One of the interesting features of this model is that it has just one extra parameter and fits the data as closely as the exponential cut off model with two extra parameters which is discussed in section (2.6).

2.4 Model 4: pre-inflationary radiation domination (PIR)

In this model we take into account the effect of a pre-inflation radiation-dominated era which can lead to modulations in the primordial power spectrum [23, 24, 53]. The transition from a pre-inflation radiation-dominated phase to de-Sitter universe was first studied by Vilenkin & Ford [53]:

$$\mathcal{P}(k) = A_s \, k^{1-n_s} \, \frac{1}{4y^4} \left| e^{-2iy}(1 + 2iy) - 1 - 2y^2 \right|^2 , \tag{2.22}$$

where $y = k/k_c$. The cut off scale k_c is set by the Hubble parameter and is proportional to k^2. Here, current horizon crosses the horizon around the onset of inflation. This model produces cut off followed by a bump like feature in the primordial power spectrum.

2.5 Model 5: pre-inflationary kinetic domination (PIK)

We also consider a model given in [21] which also produces cut off due to possible existence of a kinetic stage in the pre-inflationary era, where the velocity of the scalar field was not negligibly small. In order to affect the low-ℓ multipoles, this stage should occur very close to the beginning of the last 65 e-fold period of inflation. If scales corresponding to the current horizon have exited the horizon around the onset of inflation then this could cause a significant drop on the large angular scales of the primordial spectrum. Here the inflation potential is quadratic

$$V(\phi) = \frac{m_\phi^2 \phi^2}{2}, \tag{2.23}$$

with initial conditions given by $\phi_{\rm in} = 18 M_p$, $(d\phi/dt)_{\rm in} \simeq -m_\phi \phi_{\rm in}$. The form of primordial perturbations for pre-inflationary kinetic domination model can be expressed as

$$\mathcal{P}(k) \; = \; \frac{H_{\rm inf}^2}{2\pi^2} \, k \mid A - B \mid^2, \tag{2.24}$$

where

$$A = \frac{e^{-ik/H_{\rm inf}}}{\sqrt{32\,H_{\rm inf}/\pi}}\left[\mathcal{H}_0^{(2)}\left(\frac{k}{2H_{\rm inf}}\right) - \left(\frac{H_{\rm inf}}{k} + i\right)\mathcal{H}_1^{(2)}\left(\frac{k}{2H_{\rm inf}}\right)\right],$$
$$B = \frac{e^{ik/H_{\rm inf}}}{\sqrt{32\,H_{\rm inf}/\pi}}\left[\mathcal{H}_0^{(2)}\left(\frac{k}{2H_{\rm inf}}\right) - \left(\frac{H_{\rm inf}}{k} - i\right)\mathcal{H}_1^{(2)}\left(\frac{k}{2H_{\rm inf}}\right)\right],$$

$H_{\rm inf}$ denotes the Hubble parameter in physical units during inflation, $\mathcal{H}_0^{(2)}$ and $\mathcal{H}_1^{(2)}$ denote the Hankel function of the second kind with order 0 and 1, respectively. Here the cut off is proportional to k^3. However, this model has a scale invariant primordial power spectrum for large k and therefore is strongly disfavored by the current data, despite producing low CMB power.

If we consider that quantum fluctuations originate in the Bunch-Davies vacuum as is considered in [23] also,[1] then the primordial power spectrum can be rewritten as:

$$\mathcal{P}(k) \;=\; A_s'\left(\frac{k}{k_0}\right)^{n_s-1}\frac{H_{\rm inf}^2}{2\pi^2}\,k\,|\;A - B\;|^2, \tag{2.25}$$

with

$$A_s = A_s'\frac{H_{\rm inf}^2}{2\pi^2}\,k_0\,|\;A(k_0) - B(k_0)\;|^2. \tag{2.26}$$

where k_0 is the pivot scale.

This model also has one extra parameter $H_{\rm inf}$ which we constrain from WMAP 9 + Planck data.

2.6 Model 6: exponential cut off (EC)

The primordial power spectrum which has lesser power at low k can also be approximated by imposing an exponential cut off at $k < k_c$ [21, 70–72] on the power law model $\mathcal{P}_o(k)$:

$$\mathcal{P}(k) \;=\; \mathcal{P}_o(k)\left[1 - e^{-(k/k_c)^\alpha}\right], \tag{2.27}$$

where α is a measure of the steepness of the cut off. On small angular scales, this parameterization behaves as simple power law model and qualitative features in the CMB power spectrum that determine the constraints on the cosmological parameters are not affected except at the low-ℓ multipoles.

2.7 Model 7: Starobinsky (SB)

Another model which predicts a step-like feature in $\mathcal{P}(k)$ was proposed by Starobinsky [54], which assumes that there is a sharp change in slope of potential of the scalar field $V(\phi)$ at certain ϕ_0 which controls the inflationary stage. The general form of scalar field potential which has a rapid change for such cases can be expressed as

$$V(\phi) = \begin{cases} V_0 + A_+\,(\phi - \phi_0) & \phi\; > \;\phi_0 \\ V_0 + A_-\,(\phi - \phi_0)\,, & \phi\; < \;\phi_0 \end{cases}, \tag{2.28}$$

[1] Note that the validity of imposing Bunch-Davies vacuum in Kinetic Domination regime at all scales is questionable but this assumption does provide a specific form of model PPS to study.

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where A_- and A_+ are model parameters assumed to be greater than 0. It can be found that if the width $\Delta\phi \approx (\phi - \phi_0)$ of the singularity is small enough then the resulting adiabatic primordial spectrum is non-flat around the point k_c which can be expressed analytically in terms of transfer function applied on any underlying power spectrum:

$$\mathcal{P}(k) = \mathcal{P}_o(k)\, \mathcal{D}^2(y, \Delta), \tag{2.29}$$

where the transfer function is given by [73, 74],

$$\begin{aligned}\mathcal{D}^2(y, \Delta) = \Bigg[1 &+ \frac{9\Delta^2}{2}\left(\frac{1}{y} + \frac{1}{y^3}\right)^2 + \frac{3\Delta}{2}\left(4 + 3\Delta - \frac{3\Delta}{y^4}\right)\frac{1}{y^2}\cos(2y) \\ &+3\Delta\left(1 - (1+3\Delta)\frac{1}{y^2} - \frac{3\Delta}{y^4}\right)\frac{1}{y}\sin(2y)\Bigg].\end{aligned} \tag{2.30}$$

Here $y = k/k_c$ and $\Delta = \frac{A_+ - A_-}{A_+}$. In this model we have applied transfer function on simple power law model $\mathcal{P}_o(k)$. k_c determines the location of the step and has no effect on the shape of the spectrum besides the overall normalization. For $R = A_+/A_- < 1$, there is a sharp decrease of spectrum followed by a bump at small k with large oscillations and a flat upper plateau on small scales (see also [75]). For $R > 1$ this picture is inverted and has a step-down like feature. Here, in this model for large k the modulation term becomes close to 1 and we get simple power law model.

2.8 Model 8: Starobinsky cut off (SBC)

As discussed in the previous section, Starobinsky's transfer function can be imposed on any class of primordial power spectrum. Here, in this model, we superimpose Starobinsky modulation on an exponential cut off spectrum (model 6) with a adjustable sharpness of the cut off:

$$\mathcal{P}(k) = \mathcal{P}_o(k)\left[1 - e^{-(\varepsilon\, k/k_c)^\alpha}\right]\mathcal{D}^2(y, \Delta), \tag{2.31}$$

where $\mathcal{D}^2(y, \Delta)$ is the transfer function of the Starobinsky feature described in the previous section and ε sets the ratio of the two cut off scales involved. This model has both exponential (sharp) cut off and a Starobinsky model bump like feature in the power spectrum. Previously Sinha & Souradeep [55] have found that this model provides best likelihood value among the wide range of models discussed here. For simplicity, we reduce the degree of freedom of this model by fixing ε=1. We found that this parameterization does not affect final results.

3 Methodology and parameter estimation

We employ Monte Carlo Markov Chain (MCMC) analysis to estimate the parameters of PPS models we consider for our study and use publicly available code `COSMOMC` [76, 77] for this purpose. `COSMOMC` uses publicly available code `CAMB` [78, 79] for computing angular power spectra of CMB anisotropies following a line of sight approach which was given in [80]. `COSMOMC` uses the likelihood code provided by the WMAP and Planck team for computing the likelihood.

WMAP 9 parameter estimation methodology is given in [10] which is not very different than what was outlined in [81]. WMAP 9 year likelihood code does not need any extra parameter and computes the likelihood at low and high ℓ's differently for the temperature and polarization data. At low-ℓ ($l \leq 32$) TT likelihood is computed from the angular power spectrum estimated using Gibbs sampling and at high-ℓ ($l > 32$) TT likelihood is calculated from the angular power spectrum estimated from an optimum quadratic estimator. For polarization, high-ℓ ($l > 23$) TE, EE and BB likelihoods are computed using `MASTER` and low-ℓ ($l \leq 23$), TE, EE and BB likelihoods are computed in the pixel space.

Apart from WMAP 9 year temperature and polarization data, we also consider Planck temperature data for our analysis which is also publicly available. Planck likelihood code (discussed in [14] and downloadable from [82]) has different modules to compute likelihood at low and high ℓ's. Planck likelihood code also computes likelihood for low-ℓ polarization data which it uses from WMAP 9 year data, however, we do not use that. We consider only modules which compute TT-likelihood at low and high ℓ. At high-ℓ (up to ℓ=2500), Planck likelihood code uses a code named `CamSpec` which has 14 extra parameter to take care of foreground and other systematic. At low-ℓ ($\ell \leq 49$) Planck likelihood code uses `COMMANDER`.

Since we perform joint WMAP 9 + Planck analysis of CMB power spectrum, it is important to discuss about consistency between WMAP 9 and Planck data. Within in the context of the ΛCDM model, it has been found that the values of some cosmological parameters like H_0 obtained from Planck measurements are significantly different from WMAP 9 measurements. It was shown in [83] that WMAP 9 angular power spectrum is about 2.6% higher at very high significance level at low ℓ's, however, no significant bias was found at high ℓ's. Similarly, Planck team has reported the the inconsistency of the angular power spectrum at multipoles $\ell \leq 40$ [14] and 2% difference of angular power spectra near the first first acoustic peak [84]. It was also shown by [85] that although best fit model to Planck data was consistent with WMAP 9 year data, but WMAP 9 best fit was found to be inconsistent with Planck data at 3σ. Recently [86] revisited the analysis of the WMAP 9 data and they also found found $\sim 2.5\%$ difference in WMAP 9 and Planck spectra at $\ell \geq 100$ at 3-5σ level. Although, some level of the inconsistency between cosmological measurements was found arising from the Planck 217×217GHz detector [87], but there still remains significant tension. The tension between WMAP 9 and Planck data could be due the different systematics present in the WMAP 9 and Planck data or it could signal the the failure of ΛCDM model which could have far reaching consequences.

We modify `CAMB` and `COSMOMC` so that the extra parameters of the PPS models can be incorporated and use priors as are given in table 1. Since for running `COSMOMC` we need covariance matrices also apart from prior range therefore we generate covariance matrices from a few trial runs.

The cosmological parameterization has been carried out by using the six basic parameters (baryon density "$\Omega_b h^2$", cold dark matter density "$\Omega_c h^2$", Thomson scattering optical depth due to reionization "τ", angular size of horizon "θ", spectral index "n_s" and scalar amplitude "$\ln 10^{10} A_s$") along with the parameters which describe the features in the PPS i.e., k_c, α, Δ etc.

Apart from the standard six cosmological parameters, we keep the values of the rest of the cosmological parameters constant. We have fixed the sum of physical masses of standard neutrinos "ν"=0.6 eV, effective number of neutrinos "$N_{\rm eff}$"=3.046, Helium mass fraction "$Y_{\rm He}$"=0.24 and the width of reionization 0.5. For the case of WMAP 9 year + Planck data all the nuisance parameters of `CamSpec` where fixed to the standard values given in [11, 14].

We perform a Markov Chain Monte Carlo analysis to determine the values of the model parameters that provide the best fit to the observed data from Planck and WMAP for CMB power spectrum. `Getdist` was used with the chains generated by `COSMOMC` to produce 2D contours and plots of the marginal posteriors.

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Parameter Name	Symbol	Prior Ranges
Baryon Density	$\Omega_b h^2$	0.005–0.1
Cold Dark Matter Density	$\Omega_c h^2$	0.001–0.99
Angular size of Acoustic Horizon	θ	0.5–10.0
Optical Depth	τ	0.01–0.8
Scalar Spectral Index	n_s	0.5–1.5
Scalar Amplitude	$\log 10^{10} A_s$	2.7–4.0
Hubble Parameter at Inflation	$H_{\rm inf}$ (Mpc^{-1})	10^{-2}–10^{-7}
Running Index	α_s	-1–1
Cut off Parameter	k_c (Mpc^{-1})	0.0–0.01
Cut off Steepness Parameter	α	1.0–15.0
Starobinsky Parameter	Δ	0.0–1

Table 1. Uniform prior used in parameter estimation.

		WMAP 9			WMAP 9+Planck		
Model	Parameter	Best Fit	68% Limit	$\chi^2 = -2\log\mathcal{L}$	Best Fit	68 % Limit	$\chi^2 = -2\log\mathcal{L}$
1 (PL)				7558.0160			15382.9400
2 (RN)	α	-0.012	-0.012± 0.022	7557.7340	-0.009	-0.009 ± 0.006	15380.6580
3 (SC)	$10^4 k_c$	2.9149	2.3597± 0.9809	7555.6080	3.0449	2.5653±0.8250	15378.2840
4 (PIR)	$10^4 k_c$	0.3910	0.5909±0.5324	7557.9100	0.3941	<0.4296	15382.1560
5 (PIK)	$10^4 H_{\rm inf}$	2.0846	2.07836 ± 1.0052	7556.1900	2.1485	2.0934± 0.8973	15380.0950
6 (EC)	$10^4 k_c$	2.9244	2.4876±1.1406	7555.6700	2.9780	2.7752± 0.9237	15378.6420
	α	7.6167	8.1354[NL]		9.22328	8.2764[NL]	
7 (SB)	$10^4 k_c$	1.4724	< 12.83404	7556.1760	14.641	< 14.5739	15375.7760
	Δ	0.3893	< 0.2583		0.0558	0.0696±0.0667	
8 (SBC)	$10^4 k_c$	3.1313	< 0.1839	7555.7640	2.9149	< 0.23354	15378.3460
	α	12.502	8.022[NL]		12.6627	8.1238[NL]	
	Δ	0.0037	<0.4509		0.055492	<0.2896	

Table 2. The best fit and mean values of the extra parameters of PPS models we consider for the WMAP 9 year and joint WMAP 9 year and Planck data. We find good constrains on the cut off scales k_c, however, our constrains on other parameters are poor. We were able to put upper limit on the Starobinsky parameter Δ but no limit (NL) was found for the exponential cut off parameter α.

3.1 Best fit parameters

We present the results of our analysis in terms of the best fit values and their mean values with 1-σ errors (when possible) for the parameters of the PPS models which characterize the primordial power spectrum. Since we find that the values of the rest of the cosmological parameters are within acceptable range and do not show any interesting correlation with our model parameters of PPS, so we do not present estimates for those here.

We present the estimates of model parameters for the WMAP 9 year and WMAP 9 year + Planck data in table 2 with the values of -2log likelihood (or "χ^2"). From the table we can

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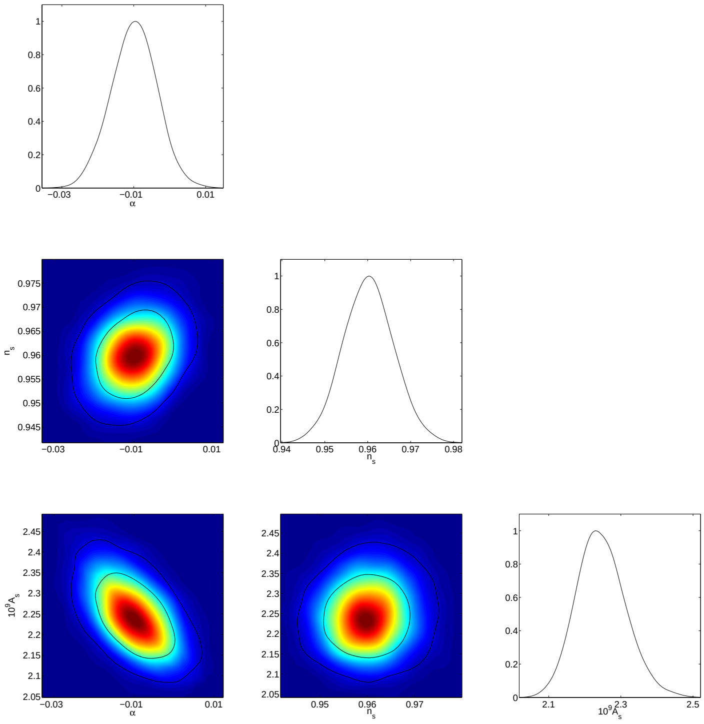

Figure 1. Two dimensional joint probability distributions and one dimensional marginal probability distribution for the parameter $10^9 A_s, n_s$ and α_s of the primordial power for RN model for WMAP 9 + Planck data.

see that all the models we consider give better fit to the data than the standard power law model. However, the improvement is marginal. We also present the ranking of the models later in this section.

One of the features which all of our models (apart from PL) have common is a cut off at large scale we have and characterized by a scale k_c. We find that cut off around $1.40 \times 10^{-4} - 3.15 \times 10^{-4}\,\mathrm{Mpc}^{-1}$ for most of the models discussed here, except for model (4) which predicts much smaller value of k_c for both data sets and model (7) which predicts much larger values of k_c for joint WMAP 9 year + Planck data set. Figures 1, 2, 3, 4, 5, 6 and 7 show the marginal one-dimensional posterior distributions and 2D contours at 68% and 95% CL of the parameters describing PPS models discussed in the work. For models (3), (6), (7), (8), we were able to obtain good bounds on the k_c for the WMAP 9 + Planck data as is clear from their two dimensional joint probability distributions. Figure 8 shows the WMAP 9 + Planck best fit primordial spectra for the range of models discussed in this work. Note that all the models we consider here have cut off at large scale $k < k_c$ and matched with the standard power law model at large k. Figures 9 & 10 show the corresponding angular power spectra C_l^{TT} obtained using best fit values of PPS parameters and other standard cosmological parameters.

3.2 Model comparison

There are many methods for model comparison and have their own advantages and disadvantages. Two of the most common methods are Akaike Information Criteria or *AIC* [88] and Bayesian Information Criteria or *BIC* [89]. *BIC* is considered more powerful than *AIC*

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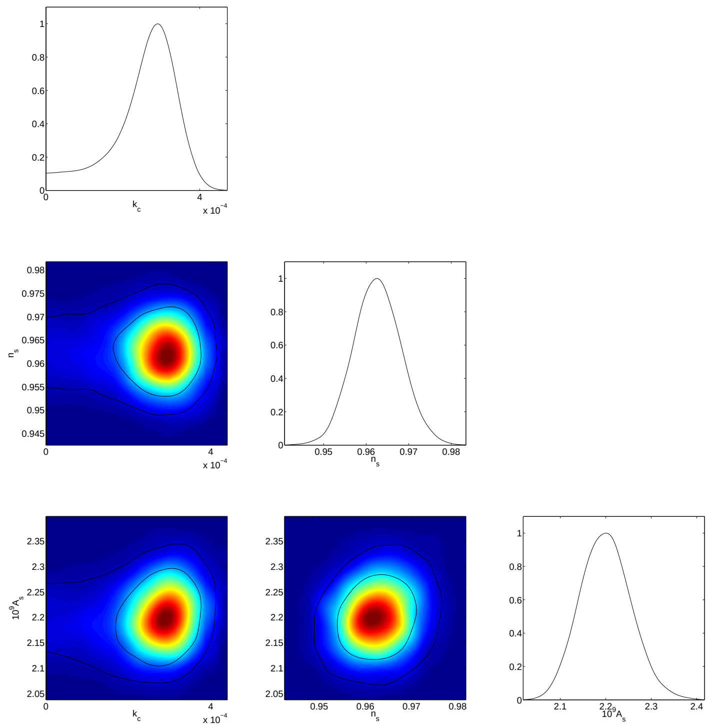

Figure 2. Same as in figure 1 for the model 3 (SC). From the figure it is clear that the cut off scale is quite well constrained from the data.

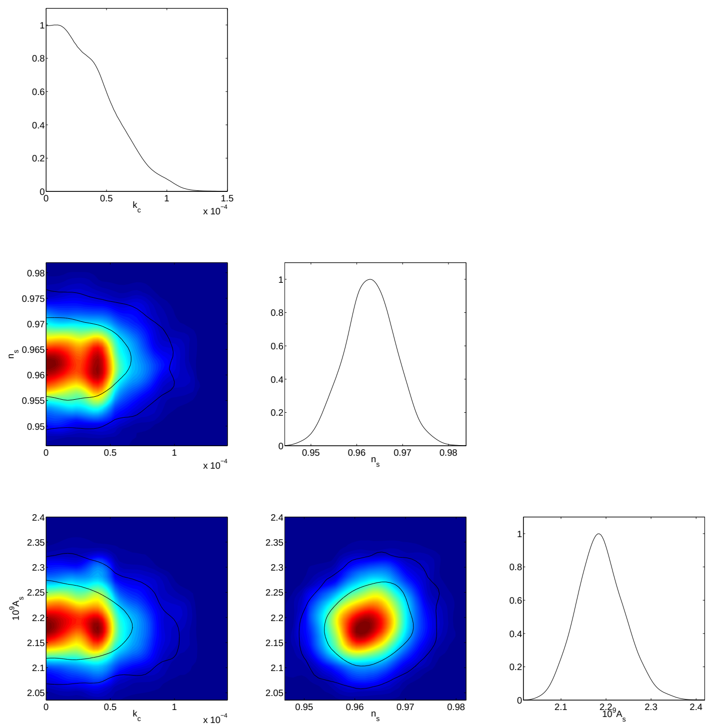

Figure 3. Marginal one dimensional probability distributions and two dimensional joint probability distribution for the model 4 (PIR) which also has just one extra parameter for the WMAP 9 + Planck data. For this model the constraints on k_c are not that good as we have for model 3 (SC) which is expected since the role of k_c in this case is slightly different.

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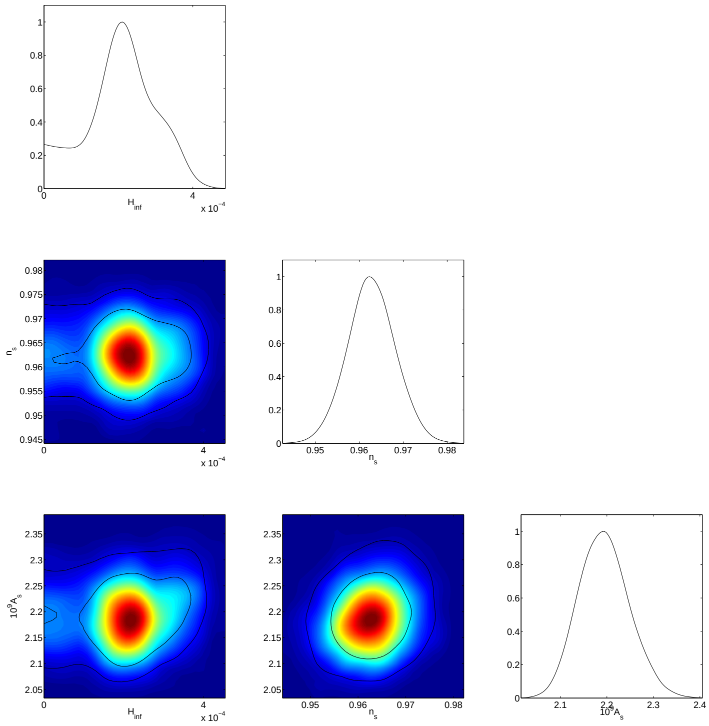

Figure 4. One dimensional marginalized probability distribution and contour plots for the parameters of PPS for model 5 (PIK) with WMAP 9 + Planck data.

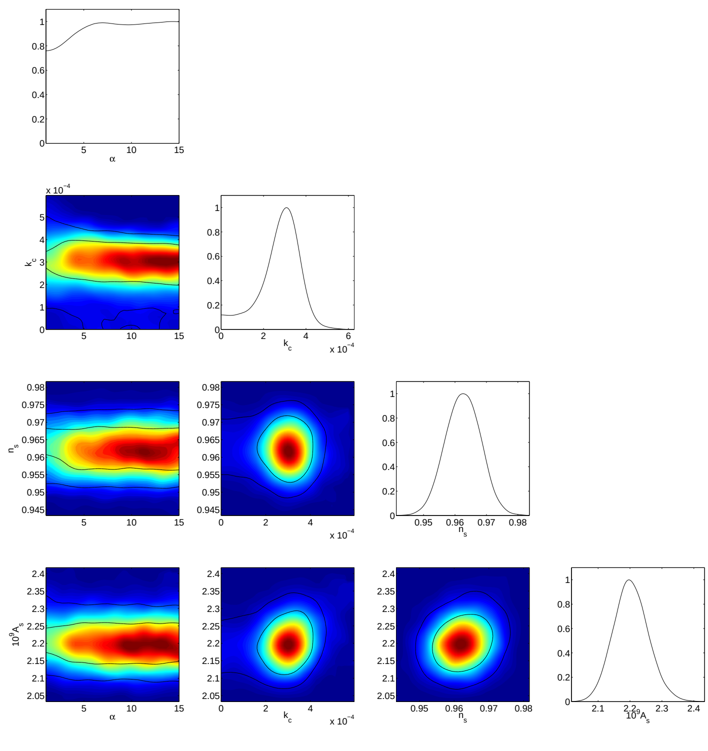

Figure 5. We find that the likelihood for WMAP 9 + Planck data is not very sensitive for parameter α for model 6 (EC) so we have poor constrains (any value of $\alpha > 5$ value is as good as any other value). However, for this model we also obtain good constrains on the cut off scale k_c.

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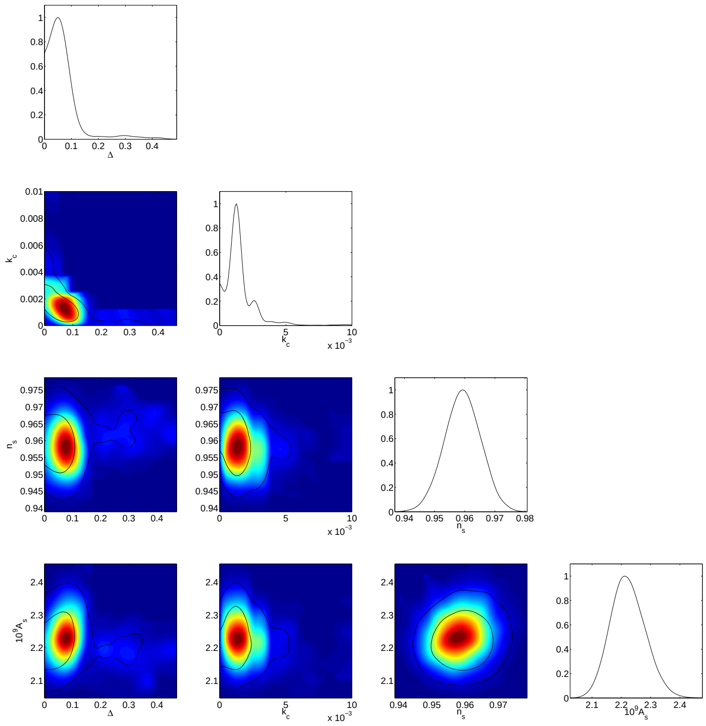

Figure 6. For model 7 (SB), we find upper limits for the fitting parameters k_c and Δ for the WMAP 9 + Planck data.

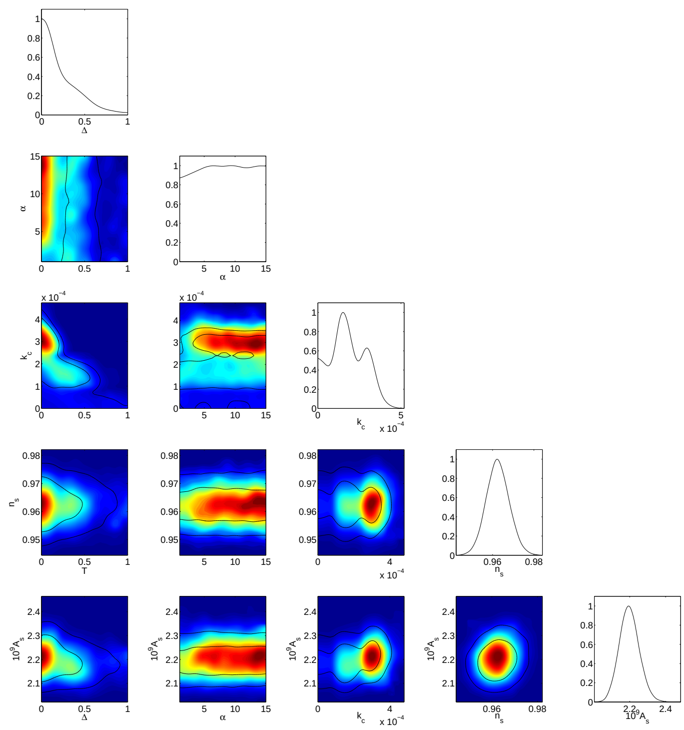

Figure 7. Like model 6 (EC), we find for model 8 (SBC) poor constraints for α, however, we find good constrains for the parameter k_c and Δ for the WMAP 9 + Planck data.

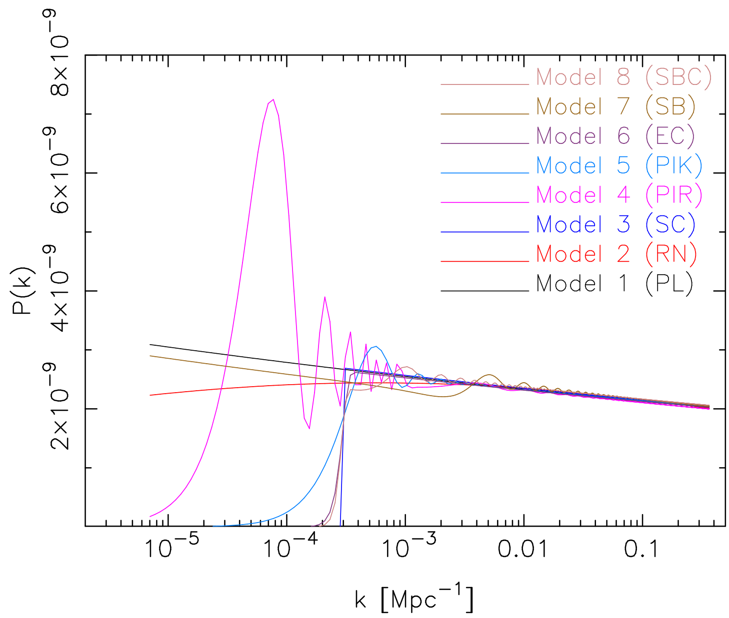

Figure 8. The best fit primordial spectra for the models consider for our analysis using WMAP 9 + Planck data. Note that all the models we considered in this work have cut off at large scale $k < k_c$ and matched with the standard power law model. All the models we consider give better likelihood than the pure power law model.

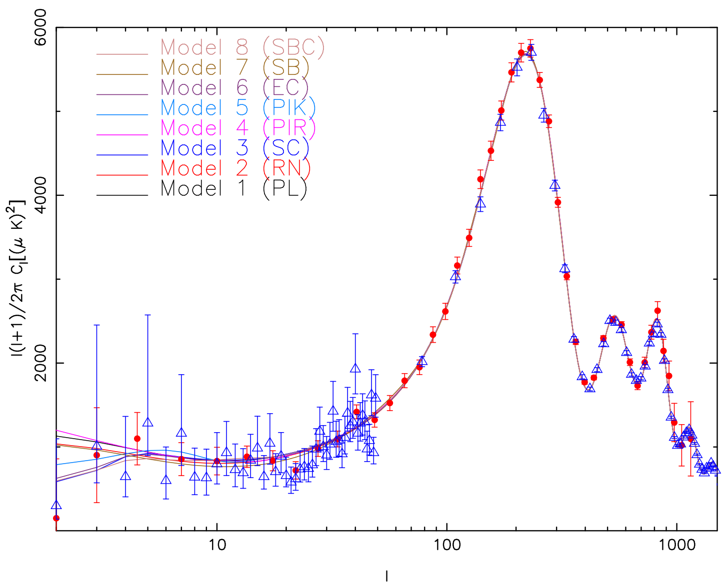

Figure 9. The best fit angular power spectra C_l^{TT} for the models of PPS we consider using WMAP 9 + Planck data. The observed data points for WMAP 9 + Planck data are also shown by red dots and blue triangles respectively with error bars.

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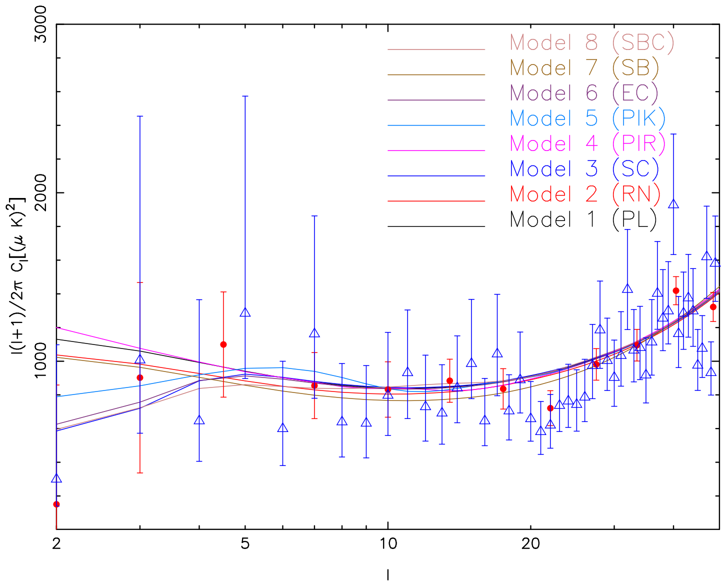

Figure 10. Same as in figure 9 at low-ℓ. This figure shows that the model 7 (SB) gives power suppression up to a large values of l so fit the Planck data better than other models.

since it explicitly takes into account the number of data points when penalizing the models with more parameters. AIC has been most commonly used [74, 90] and is defined in the following way:

$$AIC = -2 \ln \mathcal{L}(\mathbf{d}|\theta) + 2k, \tag{3.1}$$

where $\mathcal{L}(\mathbf{d}|\theta)$ is likelihood, $\mathbf{d}$ data vector, $\theta \in \mathbb{R}^k$ is the parameter vector and k is the number of parameters. The best model is one which have minimum value of AIC. BIC is also defined in the similar way:

$$BIC = -2 \ln \mathcal{L}(\mathbf{d}|\theta) + k \ln N, \tag{3.2}$$

where N is the number of data points.

The second terms in the r.h.s. of equation (3.1) and (3.2) work as regularization functions and penalize models with more number of parameters like any other regularization function does and can be related to Maximum Entropy Method [51]. BIC is more conservative than AIC since it puts stringent penalty for models with more fitting parameters for $\ln N > 2$ i.e., the penalty term exceeds $2k$. It is well known that AIC minimizes the Kullback-Leibler divergence between the estimate from a candidate model and the true distribution and BIC selects a model that maximizes the posterior probability distribution. For AIC, $\Delta AIC \equiv AIC_i - AIC_{\rm min}$ represents preference of model i over the the best fit model. Models with $\Delta AIC \leq 2$ have substantial support, models with $4 < \Delta AIC < 7$ have considerably less support and those with $\Delta AIC > 10$ have essentially no support compared to best fit model [91]. $\Delta BIC \equiv BIC_i - BIC_{\rm min}$ represents the preference of best fit model (i.e model with minimum value of BIC ($BIC_{\rm min}$)) over model i. ΔBIC values of $\Delta BIC \leq 2$, $2 < \Delta BIC \leq 6$, $6 < \Delta BIC \leq 10$ and $\Delta BIC > 10$ represent weak, positive, strong and very strong support for best fit model respectively [92]. However, in order to calculate BIC, it is not straightforward to find out the number of independent data points (N) for WMAP 9 year + Planck data as it is a correlated data set. But it is clear that as the number of data points is quite large, BIC becomes highly sensitive towards the number of extra

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	WMAP 9		WMAP 9+Planck	
Model	ΔAIC	ΔBIC	ΔAIC	ΔBIC
1 (PL)	0.408	-5.632	3.164	-10.036
2 (RN)	2.126	2.126	2.882	-3.718
3 (SC)	0.000	0.000	0.508	-6.092
4 (PIR)	2.302	2.302	4.380	-2.22
5 (PIK)	0.582	0.582	4.319	-4.281
6 (EC)	2.062	8.102	2.866	2.866
7 (SB)	2.56	8.608	0.000	0.000
8 (SBC)	4.156	16.236	4.570	11.17

Table 3. This tables shows ΔAIC and ΔBIC for the different models for WMAP 9 and WMAP 9+Planck data with respect to the model which has lowest chi-square. For WMAP 9 data we find that the sharp cut (SC) model (model 3) gives lowest AIC, however, for WMAP 9 year + Planck, Starobinsky model (SB) model (model 7) gives the lowest AIC. The reason behind model 7 being preferred by WMAP 9 year + Planck data is that it suppresses power at higher angular scales $\ell \leq 30$. However, BIC always prefers power law model over the cut off models because of least number of parameters.

parameters. For a sensible estimates of N, we can assume 1168 (WMAP 9 Master TT ℓ's) + 777 (MWAP 9 Master TE ℓ's) + 1170 (WMAP 9 TT/TE/EE/BB low-ℓ chi2 pixels) + 2499 (Planck CamSpec+commander ℓ's) = 5614 independent data points for WMAP 9 + Planck data set. Therefore, we get for WMAP 9 data $\ln(N) = 8.04$ and $\ln(N) = 8.63$ for WMAP 9 + Planck data set.

We present ΔAIC and ΔBIC values for the models we consider in table 3. We find that for the WMAP 9 year data, sharp cut off model (SC) gives the best (minimum) value of AIC but is as good as power law model, however, for WMAP 9 year + Planck data Starobinsky model (SB) has the lowest value of AIC which we believe is this due to the fact the this model gives suppression of power up a higher values of l as required by the Planck data. Using BIC rule for model selection, one finds that power law model is always favored over the cut off models. Moreover, among cut off models, model with lesser number of parameters is always favored. It is also worth mentioning that model (8) has a larger value of χ^2 despite having more parameters and therefore is disfavored by the current data.

4 Discussion and conclusions

The observed CMB power spectrum is in striking agreement with the standard ΛCDM model with almost scale-invariant adiabatic fluctuations produced during the inflationary epoch. However in such studies, some anomalies have been observed such as low CMB power on large angular scales. It has been observed that inflationary epoch cannot be well described by simple form of scalar power spectrum based on the smooth slow roll approximation and the presence of the cut off in the primordial power spectrum is essential for extension of this simplistic picture in order to explain low CMB anomaly. In this work, we have explored different parameterizations of inflationary driven primordial spectra which have cut off at large angular scales so as to describe low CMB anomaly.

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We have analyzed the complete CMB data sets of WMAP 9 year and Planck. We perform a Markov Chain Monte Carlo analysis to determine parameters that provide the best fit to the data for the CMB angular power spectrum. We find that primordial power spectrum with cut off, in general, leads to improvement of the likelihood and marginal preference for a non vanishing cut off scale of k_c in some cases. Due to the large variance in the CMB temperature at low multipoles, we could only place weak constraints on some parameters of our model like α, however, our constrains on the cut off scale are fairly good.

In order to quantify the significance of the fits we have used Akaike information criterion and Bayesian information criterion. We find that for the WMAP 9 data, among various models discussed here, model (3) which has sharp cut off provides best likelihood value, but is as good as power law model as per AIC. For the WMAP 9 year + Planck data set, we find only Starobinsky's model (7) is able to explain suppression up to multipoles $\ell \leq 30$ as is indicated by the much large value of the $k_c = 14.64 \times 10^{-4}\,\mathrm{Mpc}^{-1}$ and is slightly preferred over other models as per AIC. Although, Starobinsky's model improves the fit in the Planck CMB power spectrum in the region $\ell \leq 30$, the produced suppression is still not enough and there is some scope of improvement in the fit. However, using BIC, one finds that power law model is always preferred over all cut off models.

It is also important to note that the power suppression in the CMB anisotropy is currently a subject of intense debate. The CMB suppression could be caused by other mechanisms and therefore, it is important to improve and develop current techniques which allow us better understanding of CMB suppression. There are also models such as [93–95] which have oscillations superimposed on primordial power spectrum which also provide better fit to CMB power spectrum, but in these models the cut off is not evident as is preferred by current data and oscillations cover over whole range of k (unlike the models discussed here, where oscillations die out after the cut off). Finally, we conclude that the present motivation for the low CMB power at small ℓ with an infrared cut off is very high and there is significant scope in improving the estimates of power suppression on the basis of modeling of the primordial power spectrum with the upcoming and future data.

Acknowledgments

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