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Gravitational collapse in an expanding background and the role of substructure – II. Excess power at small scales and its effect on collapse of structures at larger scales

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ABSTRACT

We study the interplay of clumping at small scales with the collapse and relaxation of perturbations at larger scales using *N*-body simulations. We quantify the effect of collapsed haloes on perturbations at larger scales using the two-point correlation function, moments of counts in cells and the mass function. The purpose of the study is twofold and the primary aim is to quantify the role played by collapsed low-mass haloes in the evolution of perturbations at large scales; this is in view of the strong effect seen when the large scale perturbation is highly symmetric. Another reason for this study is to ask whether features or a cut-off in the initial power spectrum can be detected using measures of clustering at scales that are already non-linear. The final aim is to understand the effect of ignoring perturbations at scales smaller than the resolution of *N*-body simulations. We find that these effects are ignorable if the scale of non-linearity is larger than the average interparticle separation in simulations. Features in the initial power spectrum can be detected easily if the scale of these features is in the linear regime; detecting such features becomes difficult as the relevant scales become non-linear. We find no effect of features in initial power spectra at small scales on the evolved power spectra at large scales. We may conclude that, in general, the effect on the evolution of perturbations at large scales of clumping on small scales is very small and may be ignored in most situations.

Key words: gravitation – cosmology: theory – dark matter – large-scale structure of Universe.

1 INTRODUCTION

Large-scale structures like galaxies and clusters of galaxies are believed to have formed by gravitational amplification of small perturbations (Peebles 1980; Shandarin & Zeldovich 1989; Peacock 1999; Bernardeau et al. 2002; Padmanabhan 2002). Much of the matter in galaxies and clusters of galaxies is the so-called dark matter (Trimble 1987; Spergel et al. 2003) that is believed to be essentially non-interacting and non-relativistic. The dark matter responds mainly to gravitational forces and by virtue of larger density than the ordinary or Baryonic matter, the assembly of matter into haloes and the large-scale structure is driven by gravitational instability of initial perturbations. Galaxies are believed to form when gas in highly overdense haloes cools and collapses to form stars in large numbers (Hoyle 1953; Binney 1977; Rees & Ostriker 1977; Silk 1977). Evolution of density perturbations due to gravitational interaction in a cosmological setting is, therefore, the key process for the study of large-scale structure and its evolution, and a very important one in the formation and evolution of galaxies. The basic equations for this are well known (Peebles 1974) and are easy to solve when the amplitude of perturbations is small. At this stage, perturbations at each scale evolve independently on perturbations at other scales, and mode coupling is subdominant. Once the amplitude of perturbations at relevant scales becomes large, the coupling with perturbations at other scales becomes important and cannot be ignored. The equation for evolution of density perturbations cannot be solved for generic perturbations in this regime, generally called the non-linear regime. One can use dynamical approximations for studying mildly non-linear perturbations (Zel'Dovich 1970; Gurbatov, Saichev & Shandarin 1989; Matarrese et al. 1992; Brainerd, Scherrer & Villumsen 1993; Bagla & Padmanabhan 1994; Sahni & Coles 1995; Hui & Bertschinger 1996; Bernardeau et al. 2002). Statistical approximations and scaling relations can be used if a limited amount of information is sufficient (Davis & Peebles 1977; Hamilton et al. 1991; Nityananda & Padmanabhan 1994; Jain, Mo & White 1995; Padmanabhan 1996; Padmanabhan et al. 1996; Peacock & Dodds 1996; Ma 1998; Kanekar 2000; Smith et al. 2003). In general, however, we require cosmological *N*-body simulations (Bertschinger 1998; Bagla 2005) to follow the detailed evolution of the system.

In *N*-body simulations, we simulate a representative region of the universe. This region is a large but finite volume. Effects of perturbations at scales smaller than the mass resolution of the simulation,

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and of perturbations at scales larger than the box, are ignored. Indeed, even perturbations at scales comparable to the box are undersampled. It has been known for a long time that the perturbations at scales much larger than the simulation volume can affect the results of *N*-body simulations (Gelb & Bertschinger 1994a,b; Tormen & Bertschinger 1996; Cole 1997; Bagla & Ray 2005; Sirko 2005; Bagla & Prasad 2006; Power & Knebe 2006; Bagla, Prasad & Khandai 2008; Ryuichi et al. 2008). It is possible to quantify these effects and even estimate whether a given simulation volume is large enough to be representative or not (Bagla & Ray 2005; Bagla & Prasad 2006). It has been shown that for gravitational dynamics in an expanding universe, perturbations at small scales do not influence collapse of large-scale perturbations in a significant manner (Peebles 1974, 1985; Little, Weinberg & Park 1991; Bagla & Padmanabhan 1997b; Couchman & Peebles 1998) as far as the correlation function or power spectrum at large scales is concerned. This has led to a belief that ignoring perturbations at scales much smaller than the scales of interest does not affect results of *N*-body simulations. Recently, we have shown that if large-scale collapse is highly symmetric, then the presence of perturbations at much smaller scales affect the evolution of density perturbations at large scales (Bagla, Prasad & Ray 2005). Here, we propose to study the effect of small scales on the collapse of perturbations at large scales in a generic situation.

Substructure can play an important role in the relaxation process. It can induce mixing in phase space (Lynden-Bell 1967; Weinberg 2001) or change halo profiles by introducing transverse motions (Peebles 1990; Subramanian 2000), and gravitational interactions between small clumps can bring in an effective collisionality even for a collisionless fluid (Ma & Bertschinger 2003; Ma & Boylan-Kolchin 2004). Thus, it is important to understand the role played by substructure in gravitational collapse and relaxation in the context of an expanding background.

Whether the evolution of density perturbations is affected by collapsed structure or not depends on the significance of mode coupling between these scales. We summarize the known results about mode coupling here.

(i) Large scales influence small scales in a significant manner. If the initial conditions are modified by filtering out perturbations at small scales, then mode coupling generates small-scale power. If the scale of filtration is smaller than the scale of non-linearity at the final epoch, then the non-linear power spectrum as well as the appearance of large-scale structure is similar to the original case (Peebles 1985; Little et al. 1991; Bagla & Padmanabhan 1997b; Couchman & Peebles 1998).

(ii) Non-linear evolution *drives* every model towards a weak attractor [$P(k) \simeq k^{-1}$] in the mildly non-linear regime ($1 \leq \bar{\xi} \leq 200$) (Klypin & Melott 1992; Bagla & Padmanabhan 1997b).

(iii) In the absence of initial perturbations at large scales, mode coupling generates power with [$P(k) \simeq k^4$] that grows very rapidly at early times (Bagla & Padmanabhan 1997b). There are a number of explanations for this feature, ranging from second-order perturbation theory to momentum and mass conserving motion of a group of particles. The k^4 tail can also be derived from the full non-linear equation for density (Zel'Dovich 1965; Peebles 1974, 1980).

(iv) If we consider large-scale perturbations to be highly symmetric, e.g. planar, then small-scale fluctuations play a very important role in the non-linear evolution of perturbations at large scales (Bagla et al. 2005).

While the effect of large scales on small scales is known to be significant, particularly if the larger scales are comparable to the scale of non-linearity, the effect of small scales on larger scales is known to be small in most situations. Even though this effect has not been studied in detail, many tools have been developed that exploit the presumed smallness of the influence of small scales on large scales (Bond & Myers 1996; Monaco, Theuns & Taffoni 2002; Monaco et al. 2002).

Considerable work has been done in recent years on the effects of the *pre-initial* conditions used in *N*-body simulations (Bagla & Padmanabhan 1997a; Joyce et al. 2005; Gabrielli et al. 2006; Marcos et al. 2006; Baertschiger et al. 2007a,b,c; Joyce & Marcos 2007a,b). We use the term pre-initial conditions to refer to the distribution of particles on which the initial density and velocity perturbations are imprinted. The pre-initial conditions are expected to have no density perturbations or symmetry, but it can be shown that at least one of these requirements must be relaxed in practice. This can lead to growth of some modes in a manner different from that expected in the cosmological perturbation theory. Our work allows us to estimate the effect such discrepant modes can have on the non-linear evolution of clustering at these scales. Our work also allows us to understand the effects that may arise if the primordial power spectrum deviates strongly from a power law at small scales.

The evolution of perturbations at small scales depends strongly on the mass and force resolution. A high force resolution can lead to better modelling of dense haloes, but gives rise to two-body collisions (Splinter et al. 1998; Binney & Knebe 2002; Binney 2004; Diemand et al. 2004; El-Zant 2006; Romeo et al. 2008). A high force resolution without a corresponding mass resolution can also give misleading results as we cannot probe *shapes* of collapsed objects (Kuhlman, Melott & Shandarin 1996). In addition, discreteness and stochasticity also limit our ability to measure physical quantities in simulations, and these too need to be understood properly (Romeo et al. 2008; Thiebaut et al. 2008). In all such cases, the errors in modelling is large at small scales. It is important to understand how such errors may spread to larger scales and affect physical quantities.

2 MODELS

In order to understand the effects of small-scale perturbations on large scales, we have simulated some models numerically. In these models, we either suppress or add extra power at small scales, with respect to our reference model (Model I). Some of the details of our cosmological simulations are as follows.

(i) The TreePM code (Bagla 2002; Bagla & Ray 2003) for cosmological *N*-body simulations.

(ii) 200^3 particles in a volume of 200^3 cubical cells for each simulation.

(iii) A softening length of 0.5 times the average interparticle separation in order to suppress two-body collisions.

(iv) $P(k) = Ak^{-1}$ as the reference model (Model I). This was normalized so that $\sigma^2(r = r_{nl},\, a = 1) = 1$ where $r_{nl} = 12$ grid lengths.

(v) Einstein–de Sitter universe.[1]

[1] Non-linear gravitational clustering is not likely to have a strong dependence on the choice of cosmology. By restricting ourselves to the Einstein–de Sitter universe, we have an additional check on simulations in the form of self-similar evolution of clustering for the reference model.

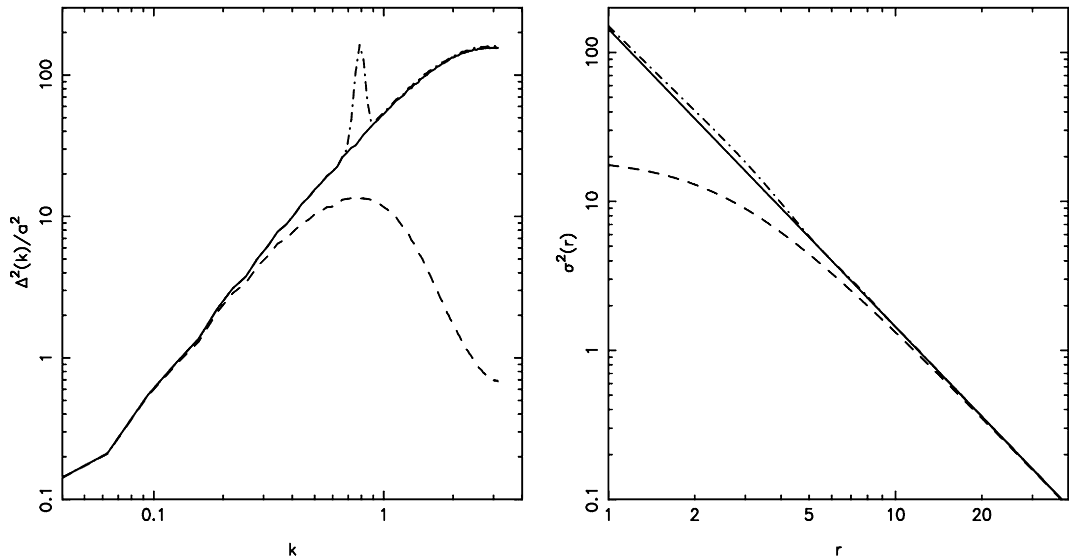

Figure 1. The left- and right-hand panels in this figure show the linearly extrapolated power spectrum $\Delta^2(k)$ and variance $\sigma^2(r)$ at the epoch $a = 1$. In both the panels, Models I, II and III are represented by the solid, dashed and dot-dashed lines, respectively.

We studied the following modifications of the reference spectrum.

(i) Model II: Gaussian truncation of the reference power spectrum at small scales, $P(k) = Ak^{-1}\exp[-k^2/k_c^2]$. We chose $k_c = k_{nyq}/4$, so that truncation is mainly at scales that are smaller than the scale of non-linearity at late times. A is chosen to be the same as for Model I.

(ii) Model III: a spike is added to the reference power spectrum, $P(k) = Ak^{-1} + \alpha A k_c^{-1}\exp[-(k-k_c)^2/2\sigma_k^2]$. We chose same k_c as in Model II, $\sigma_k = 2\pi/L_{\rm box}$ is the same as the fundamental wavenumber and we took $\alpha = 4$. A is chosen to be the same as for Model I.

Clearly, these models have additional or truncated power at small scales as compared to the reference model, while large scales are the same in all the models. The left-hand and right-hand panels in Fig. 1 show the linearly extrapolated power spectrum $\Delta^2(k)$ which we start within N-body simulations and the theoretical mass variance $\sigma^2(r)$, respectively, for the three models being considered at the last epoch, i.e. $a = 1$. From both the panels of Fig. 1, we see that all the three models have identical power at the scales much larger than the scale at which we add or suppress the power, i.e. $2\pi/k_c$.

We choose to work with the Einstein–de Sitter universe as the background, as effects of mode coupling are more important in the non-linear regime and we do not expect the cosmological parameters to influence the evolution of perturbations at these scales. The specific choice of Einstein–de Sitter universe is useful as power-law initial conditions, e.g. the reference model, are expected to evolve in a self-similar manner, and this provides a useful check for errors creeping in due to the effects of a finite box-size or other numerical artefacts.

3 RESULTS

Our goal is to understand the effects of variations in the initial power spectrum at small scales on other scales. For this, we study the three models at two representative epochs: one where the scale of modification is linear and the second epoch when the scale of non-linearity is larger than the scales where the power spectra differ from each other. We refer to these epochs as an early epoch and a late epoch. The scale of non-linearity in the reference model at the early epoch is 4.8 grid lengths and the corresponding scale at the late epoch is 12 grid lengths. The wavenumber k_c corresponds to eight grid lengths and becomes non-linear at an intermediate epoch.

Fig. 2 shows the distribution of particles in a thin slice from simulations of the three models. The left-hand column shows the distribution at the early epoch whereas the right-hand column shows the same slice at late times. The middle row shows the reference model (Model I), the top row is for model with less power at small scales (Model II) and the bottom row is for the model with excess power at small scales (Model III). The large-scale distribution of particles is similar in all the three models for both epochs, although there are significant differences at small scales. Differences are more prominent between Model II and the other models, whereas the differences between Models I and III are less obvious. Also, differences between the models diminish as we go from the early epoch to the late epoch.

Fig. 3 compares the models in a more quantitative manner. We have plotted the amplitude of clustering $\bar{\xi}(r)$ as a function of r for the three models at an early epoch (top-left frame) and at a later epoch (top-right frame). The differences between the amplitude of clustering are more pronounced at the early epoch, though even here the differences are much smaller than those seen in Fig. 1 where the linearly extrapolated $\sigma^2(r)$ has been plotted. At late times, Models I and III have an indistinguishable $\bar{\xi}(r)$, whereas Model II has a slightly smaller amplitude of clustering at small scales when compared with these two models. At very large r compared to the scale of modification, all models have the same $\bar{\xi}$ even at the early epoch.

Second row in Fig. 3 shows S_3 as a function of scale for the three models. As before, the left-hand panel is for the early epoch and the right-hand panel is for the late epoch. At large scales, larger than the scale of modification (eight grid lengths), the three models agree well. There are significant differences at small scales, particularly at the early epoch. Model II has the highest skewness, whereas Model III has the smallest skewness at small scales. This ranking does not change with time, though the differences between models decrease with further evolution of the system.

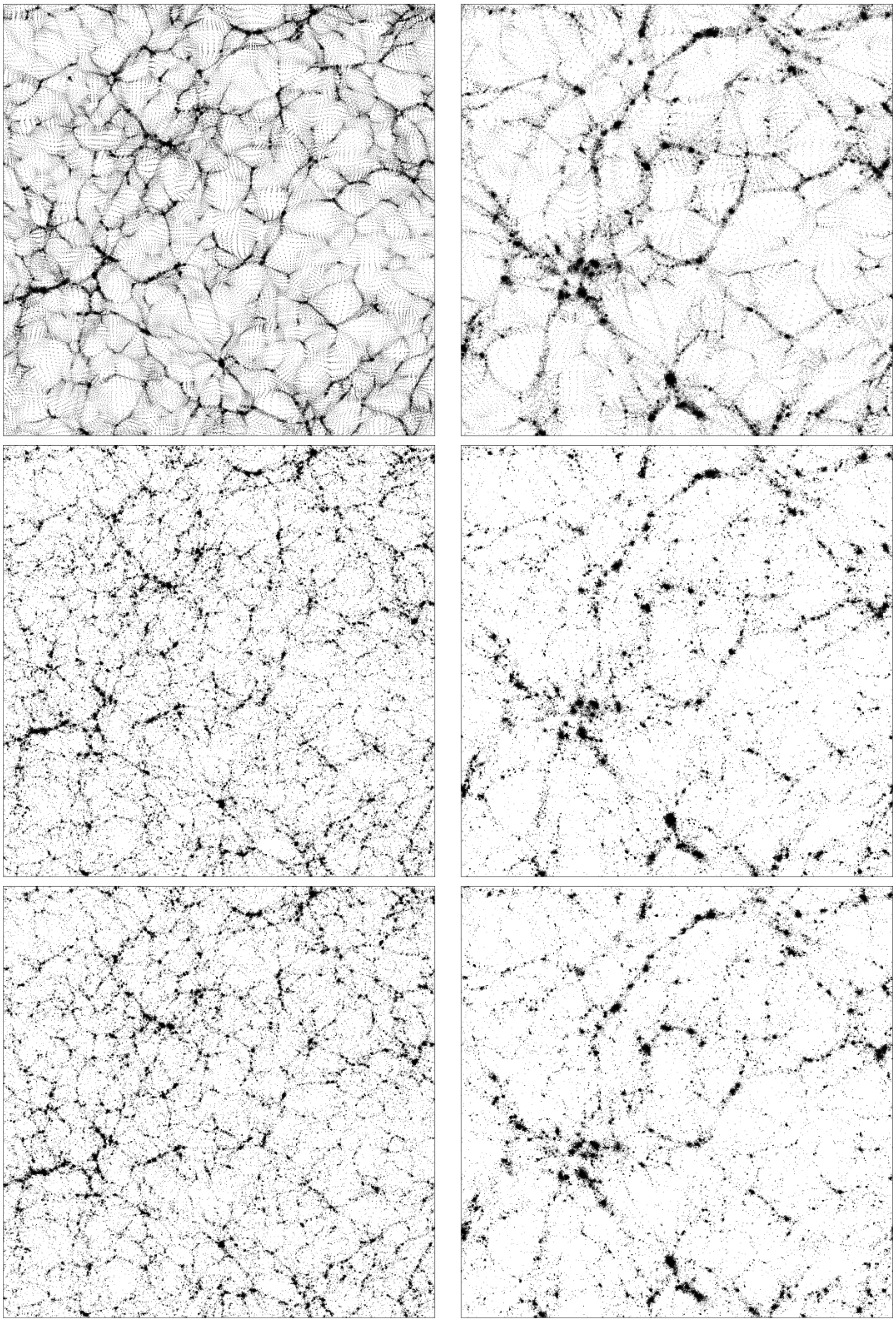

Figure 2. The first, second and third rows in the figure show the slices for the Models II, I and III respectively, at an early (first column) and a later epoch (second column). The early epoch is identified with an epoch when the scale at which we add or truncate power, i.e. $2\pi/k_c$, is linear in Model I and the later epoch is identified with an epoch when the scale $2\pi/k_c$ is non-linear in Model II.

The bottom row in Fig. 3 shows the number density of haloes as a function of mass. Mass here is shown in units of mass of each particle. Haloes were identified using the friends-of-friends (FOF) algorithm with a linking length of 0.1. We chose this linking length in order to avoid identifying smooth filaments in Model II as haloes. Haloes with a minimum of 20 particles were considered for this plot. We find that Model III has the largest number of haloes around the scale of modification, whereas Model II has the least number of haloes at this scale. Indeed, at the early epoch, Model II has a much lower number of haloes at all mass scales when compared with Models I and III. At late times, Model II continues to have fewer small mass haloes though it almost matches the other two masses at larger masses.

We find that the two-point correlation function does not retain any information about differences in initial conditions after the scale where such differences are present becomes sufficiently non-linear.

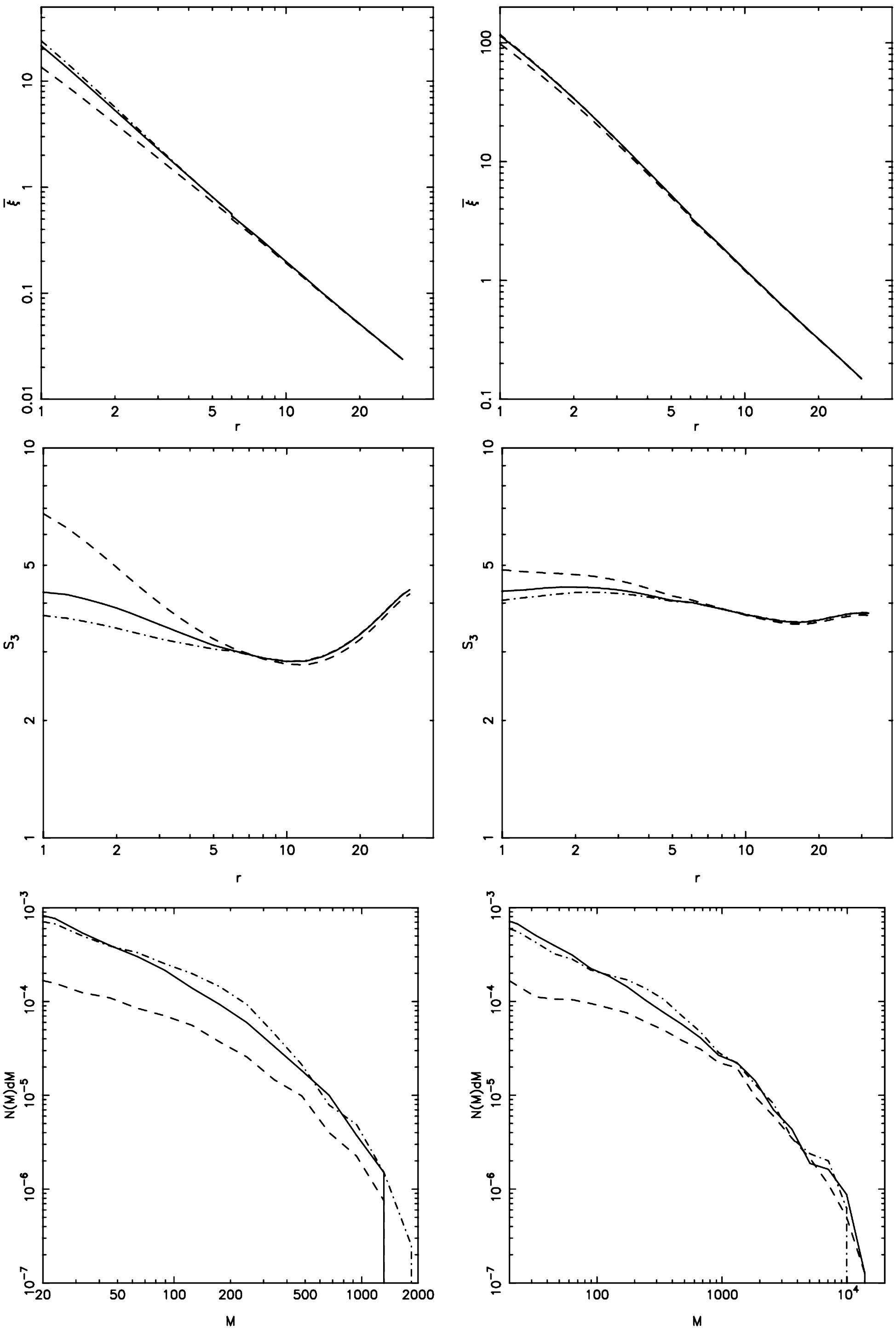

Figure 3. The first, second and third rows in this figure show the average two-point correlation function $\bar{\xi}$, skewness S_3 and comoving number density of haloes $N(M)dM$, respectively, at an early (first column) and at a later epoch (second column). Different models in all the panels are represented by the same line styles as in Fig. 1.

This is in agreement with results of the earlier studies (Peebles 1985; Little et al. 1991; Klypin & Melott 1992; Bagla & Padmanabhan 1997b; Couchman & Peebles 1998).

Skewness is a slightly better indicator than the two-point correlation function, in that it retains some information about the missing power at small scales in Model II even after the cut-off scale becomes non-linear. It does not retain much information about the excess power that is added at small scales in Model III. One possible reason for this is that the cut-off affects the shape of the power spectrum at $k \ll k_c$, whereas the effect of adding extra power is more localized. We may conclude that skewness is able to retain information about a cut-off in the initial power spectrum if the cut-off scale is not strongly non-linear. This may not have implications for observational signatures of a cut-off as observations of galaxy clustering are restricted to the redshift space and it has been shown that the redshift space distortions in the non-linear regime erase differences between models (Bagla & Ray 2006).

The number density of haloes at scales comparable to and smaller than the cut-off is smaller than that in the other models, even after the cut-off scale becomes non-linear. The mass function appears to be the most sensitive indicator of a cut-off in the power spectrum in the mildly non-linear regime.

4 DISCUSSION

We find that the memory of localized variations of initial conditions is erased in the quasi-linear regime. This erasure is almost complete in measures of the second moment, e.g. the two-point correlation function shown here. We have checked that the same is true of the power spectrum and rms fluctuations in mass. This loss of information has been pointed out in earlier work (Little et al. 1991; Bagla & Padmanabhan 1997b). The skewness appears to be a better indicator of a cut-off in the initial power spectrum, at least in the quasi-linear regime. We find that the skewness for Model II is distinctly higher than that for Model I or III, even when the scale of non-linearity is much larger than the cut-off scale. Number densities of haloes are a very faithful indicator of the cut-off, even at late times. This is to be expected given that the number density of haloes can be predicted fairly accurately using the Press–Schechter mass function (Press & Schechter 1974) that relies only on the initial power spectrum.

The question now arises as to how may we interpret these results. Here, we would like to recall the key conclusion of Paper I in the present series (Bagla et al. 2005). In Paper I, we studied the collapse of a plane wave with varying amount of collapsed haloes at a much smaller scale (as compared to wavelength of the plane wave). We found that the thickness of pancake that forms due to collapse of the plane wave is smaller if collapsed haloes are present. The reason for smaller thickness is that the gravitational interaction of infalling clumps takes away some of the longitudinal momentum and leads to an increase of the transverse momentum. Thinner pancakes imply a higher density, and clumps are able to grow very rapidly in such an environment. We were motivated to study the collapse of a plane wave, as it is known from the Zel'dovich approximation (Zel'Dovich 1970) that locally generic collapse is planar leading to the formation of pancakes.

In case of generic initial conditions that we consider here, there is no fixed large scale that is collapsing as we have perturbations at all scales. However, we have ensured that the perturbations at large scales are the same in all the three models. In this case, the effect of power on large scales is to cause collapse around peaks of density or, equivalently, empty the voids. The latter picture is more attractive as it also tells us why collapse of perturbations at a scale leads to the enhancement of power at smaller scales without any loss of power at the original scale: emptying under dense regions simply puts more and more matter in thin walls around the void that forms. We can say that the power is transferred from the scale of perturbation, that essentially is the radius of the void that forms, to the scale of thickness of pancakes surrounding the void. As a given scale becomes non-linear, we begin to see voids corresponding to this scale. Matter that collapsed at an earlier stage gets pushed into the pancakes surrounding the voids. This information about the shape of the initial power spectrum at scales smaller than the scale of non-linearity is mostly restricted to the distribution of matter within pancakes. This, in our view, explains the erasure of memory of initial conditions for the two-point correlation function. The mass function and skewness are more sensitive to the arrangement of matter within pancakes, and hence these remain different for the model with a cut-off. Once the scale of cut-off becomes strongly non-linear, most of the perturbations at this scale are expected to be part of highly overdense haloes. At this stage, we expect that all the indicators of clustering will lose information about the details of the initial power spectrum at this stage.

In Models I and III, there is significant amount of initial power at small scales. This leads to a fragmented appearance of pancakes, and clearly pancakes cannot be thinner than the clumps. In Model II, there is no initial power at small scales. Power is generated at these scales by collapse of larger modes, power grows very rapidly at small scales and the non-linear power spectrum in this case catches up with the power spectrum for the other two models.

Model III has significantly more power as compared with the reference model at small scales. This leads to a more rapid growth of perturbations at these scales, as is seen in the number density of collapsed haloes at the relevant scales at early times. At late times, these haloes are assimilated into bigger haloes, and we rapidly lose any signatures of the excess power. We expect the excess power to lead to thinner pancakes, motivated by conclusions of Paper I. However, the scale of pancakes is such that this feature is not apparent.

Another approach towards understanding the lack of effect of variations in power spectrum at small scales on larger, non-linear scales is based on the equation for evolution of density contrast (Peebles 1974). It has been shown that the leading order effect of virialized haloes on modes at much larger scales vanishes at the leading order. In case of arbitrary motion of a group of particles, a k^4 tail is generated in the power spectrum at $k \to 0$ if there is no initial power at these scales. In general, the influence of motions of particles at small scales to density perturbations is limited due to the k^2 behaviour[2] of the mode coupling terms in equations that describe the evolution of density contrast for a system of particles (Peebles 1974). The magnitude of the mode coupling at this order is proportional to the departure from virial equilibrium for the system of particles.

It is possible to consider the equations and compute the leading order contribution of interacting haloes. Clearly, this also must scale as $\mathcal{O}(k^2)$ for density contrast, but it is instructive to see if we can quantify the level to which the internal structure of clusters matters for coupling of the density fluctuations. A detailed calculation and analysis of this is presented in a forthcoming manuscript (Bagla, in preparation). We summarize a few key points here.

(i) It can be shown that the leading order contribution to mode coupling for interacting haloes comes from the halo–halo interaction where the haloes may be assumed to be point masses.

(ii) There is a next to leading order contribution due to tidal interaction of clusters.

(iii) The two contributions scale as k^2, making the contribution to power spectrum as $\mathcal{O}(k^4)$.

(iv) The magnitude of the leading order term is proportional to the departure from virial equilibrium for haloes treated as point masses.

The treatment can be generalized to an arbitrary number of haloes, and we can also study the effects of coupling between a cluster and the large-scale density distribution. These conclusions explain the results of our numerical experiments, and give a reason as to

[2] A k^2 dependence in density contrast translates into a k^4 dependence in the power spectrum.

why gravitational clustering in an expanding universe appears to be *almost* renormalizable.

5 SUMMARY

Results presented in the preceding section show that for a hierarchical model, there is little effect of features at small scales (high wavenumber) in power spectrum on collapse of perturbations at larger scales. At the same time, we see that the effect of features can be seen in several statistical indicators at the scales of features and also at smaller scales. The key conclusion that we can draw is that if we modify the power spectrum at small scales, there is no discernable effect of these modifications at larger scales. This has implications in several situations.

(i) Cosmological *N*-body simulations start with initial conditions that do not sample the power spectrum at large wavenumbers. In typical simulations of this type, a grid is used to generate initial conditions and only modes up to the Nyquist wavenumber are sampled. Indeed, if the number of particles is smaller than the number of grid cells used for generating initial conditions, the effective upper limit to wavenumbers is even more restricted (Bagla & Padmanabhan 1997a). The missing part of the power spectrum does not have any impact on the evolution of non-linear structures at scales larger than the cut-off scale. We expect the effects of missing modes at large wavenumbers to be less and less relevant as larger length scales (smaller wavenumbers) become non-linear.

(ii) It has been pointed out that the choice of pre-initial conditions and the epoch at which the initial conditions are set up can lead to spurious growth of some modes (Joyce et al. 2005; Gabrielli et al. 2006; Marcos et al. 2006; Baertschiger et al. 2007a,b,c; Joyce & Marcos 2007a,b). Clearly, these effects must be suppressed as the modes with spurious growth become non-linear.

(iii) Generation of perturbations in the early universe and their evolution towards the end of the inflationary phase can lead to a scale-dependent evolution of modes (Malquarti, Leach & Liddle 2004; di Marco et al. 2007). Our work clearly shows that such features will be impossible to detect if these are at scales that are strongly non-linear and difficult to detect if these are at scales that are mildly non-linear. If scales where such variations occur are already non-linear, then these variations do no affect collapse of larger scales. Of course, if the scales where such variations occur are linear, then these can be probed using galaxy clustering.

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